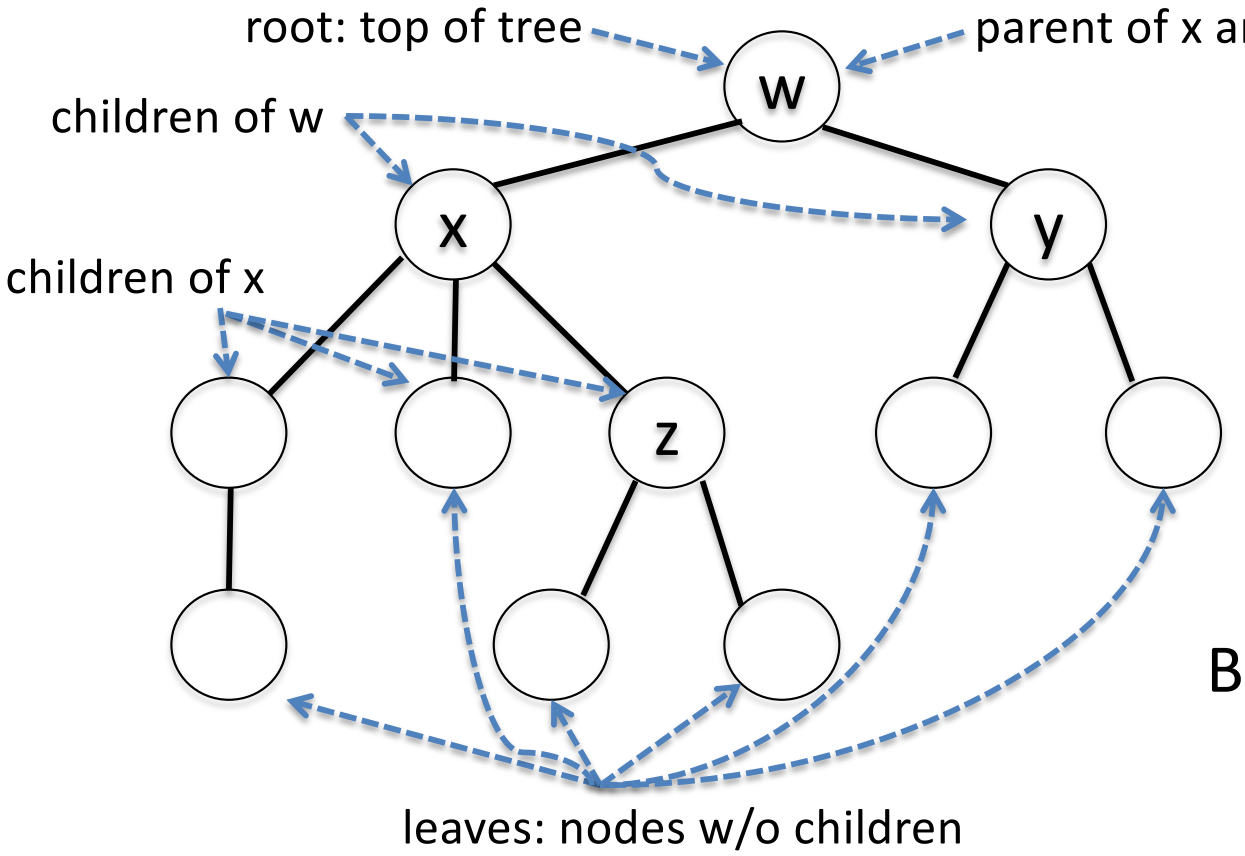
More trees

5/1/26

Administrivia

- HW 5 (sorting and search order) due tonight (5/1)

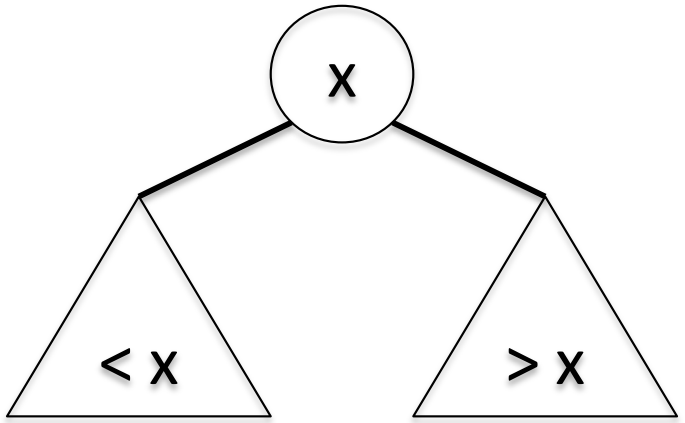
Recall: Trees



Other familial relationships:

sibling, grandparent, grandchild,
uncle (or aunt), ancestor, descendant

Binary Search Tree (BST)

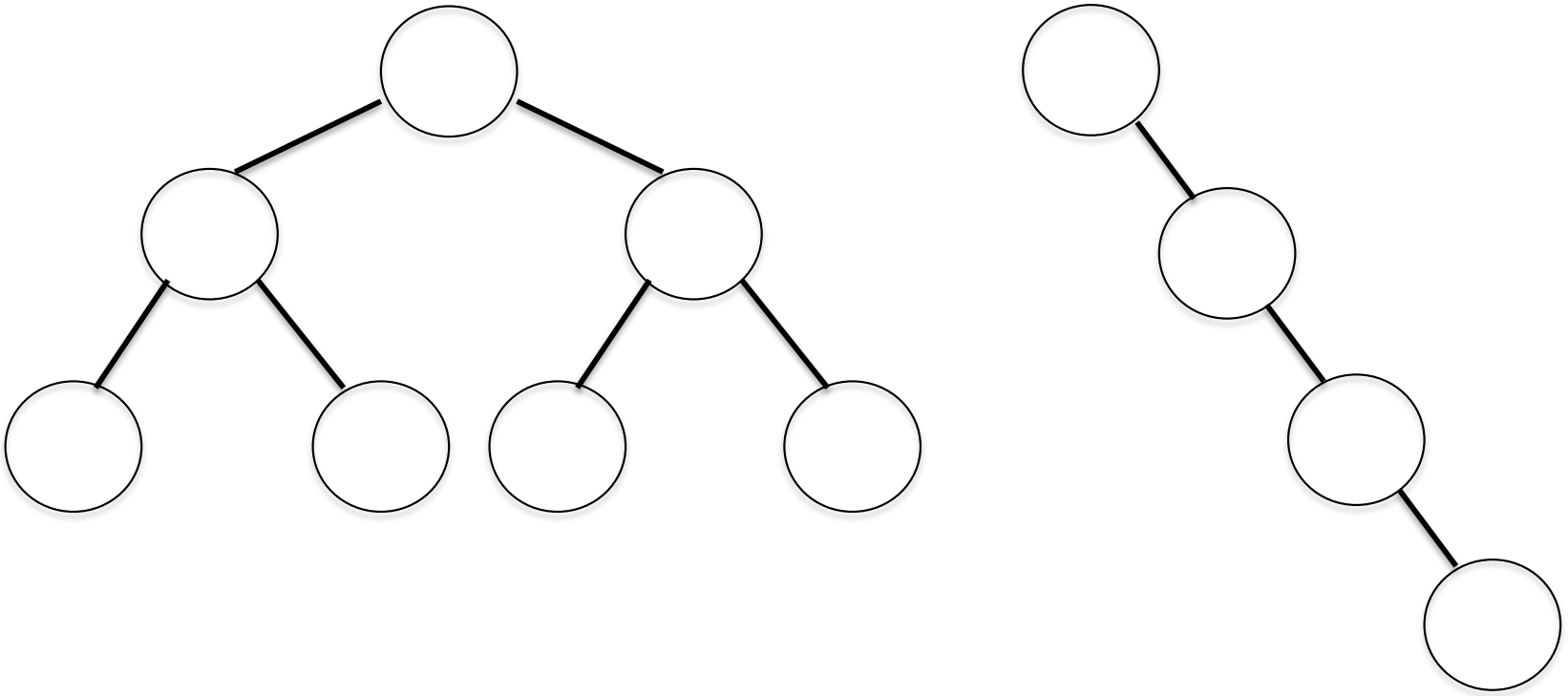


Other tree terminology

(book differs slightly; they count nodes)

- The *depth* of a node is the number of edges between it and the tree's root
- The *height* of a tree is the maximum number of edges from the root to a leaf

O(log n) ≤ height of n-node BST ≤ n-1



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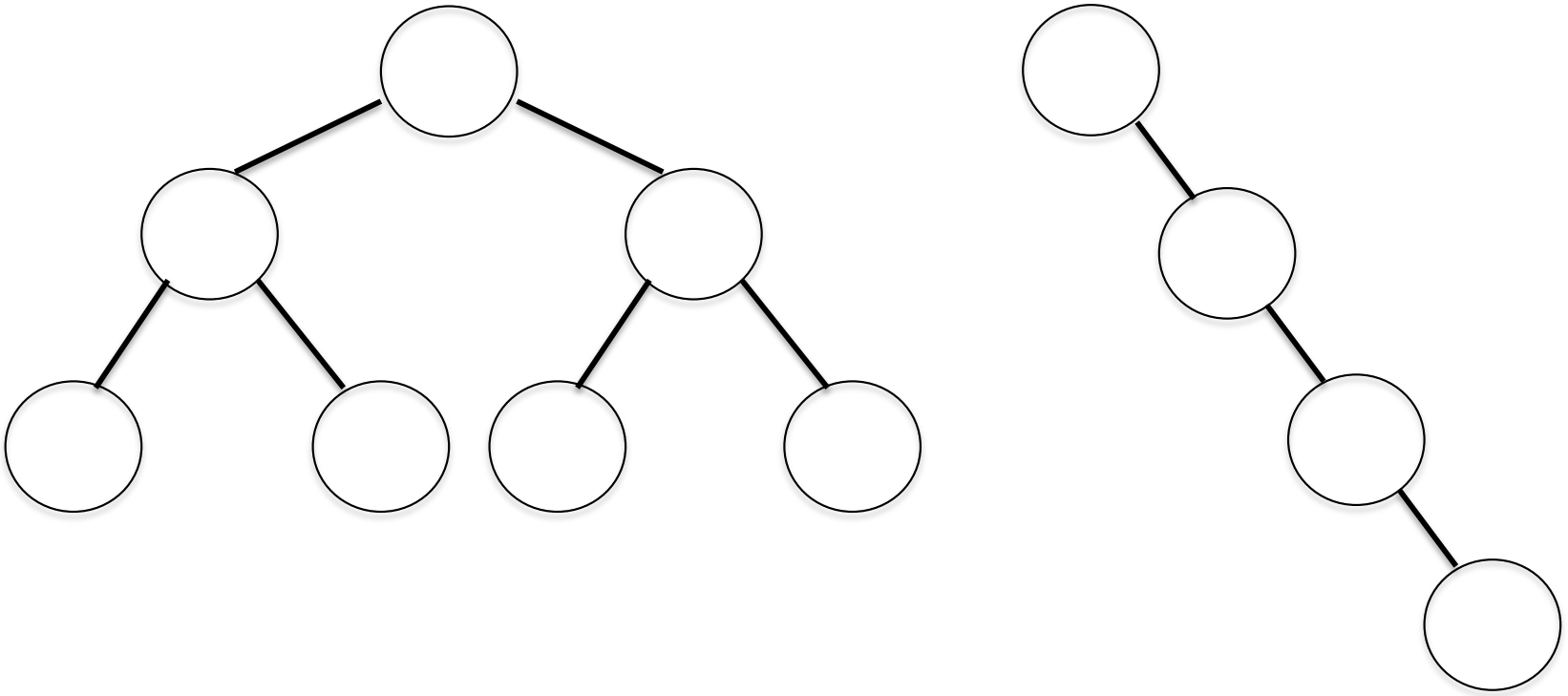

A tree is *balanced* if its height is O(log n)

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- Replace that null with the new node

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C. O(tree height)
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E. None of the above

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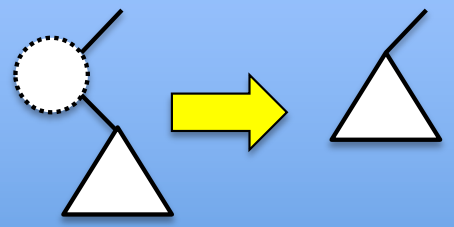

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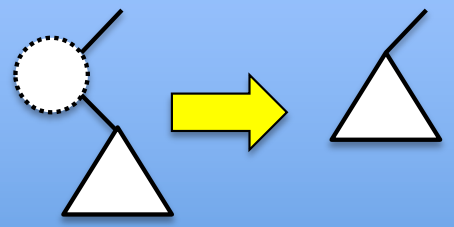

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- Case 2: This node has 2 non-null children
  - Replace the key with its *in-order successor* (next key in sorted order)
  - Remove the successor

How can we advance a Node reference to the next key?

- Case 1: Node has right subtree
  - Go one step right and then left as far as possible

- Case 2: Right subtree is null
  - Go up tree until reaching a node whose left child is the ancestor of your starting place

    (requires parent references in the Node)
  - If you try to go up from the root, there is no next key

What is the running time of remove in a balanced BST?

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B. $O(\log n)$

C. $O(\log^2 n) = O((\log n)^2)$

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Recall: Desired properties

- Ability to quickly “jump” later in the sequence of keys (like binary search)

- Ability to easily add keys into the appropriate position (like linked lists)