More sorting

4/24/26

Administrivia

- HW 4 (list methods and running times) due Tuesday

Where we were

- Sorting: putting values in order
  - In general, use a Comparator for the order (we're just going to sort integers)
  - Assume they're in an array
- Selection sort
  - Repeatedly find smallest value and put it in front
  - $O(n^2)$

Insertion sort

(code adapted from Wikipedia)

- Keep sorted portion of the array and grow it by inserting new values into the proper place

```
for(int i=0; i < A.length; i++) {        //add value A[i]
    int j = i;                           //iterator to seek possible places
    while((j > 0) && (A[j-1] > A[j])) {
        swap A[j] and A[j-1];
        j--;
    }
}
```

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What most accurately characterizes the running time of insertion sort?

A. O(1)
B. O(n)
C. $O(n^2)$
D. $O(n^3)$
E. None of the above

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Bubble sort

(code adapted from Wikipedia)

- Repeatedly find out-of-order pairs and swap them

```
boolean swapped = true;
while(swapped) {
   swapped = false;
   for(int i=0; i < A.length-1; i++)
        if(A[i] > A[i+1]) {
            swap A[i] and A[i+1];
            swapped = true;
        }
}
```

Bubble sort

(code adapted from Wikipedia)

- Repeatedly find out-of-order pairs and swap them

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while(swapped) {
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What most accurately characterizes the running time of bubble sort?

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C. $O(n^2)$
D. $O(n^3)$
E. None of the above

Bubble sort

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C. $O(n^2)$
D. $O(n^3)$
E. None of the above

Which of the sorting algorithms could cause the execution shown below?

5	1	4	2	3
1	5	4	2	3
1	2	4	5	3
1	2	3	5	4

A. Selection sort
B. Insertion sort
C. Bubble sort
D. None of these
E. More than one of these

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B. Insertion sort
C. Bubble sort
D. None of these
E. More than one of these (insertion & bubble)

Can we do better than $O(n^2)$ time?

Merge sort

- Split array into two equal-sized pieces, recursively sort each half, and merge them back

```
Mergesort(A[1..n]) {
    if(n > 1) {
        m = (n+1)/2;
        copy 1st m values of A into array L and rest into array R;
        Mergesort(L);
        Mergesort(R);
        Merge(L, R, A);
    }
}
```

```
Merge(A[1..n₁], B[1..n₂], R[1..(n₁+n₂)]) {
    //merge sorted arrays A and B into R
    int i = 1, j = 1;            //i is position in A, j is position in B
    for(int k = 1; k <= (n₁+n₂); k++) {//k is position in R
        if(i ≤ n₁ and (j > n₂ or A[i] ≤ B[j])) {
            R[k] = A[i];
            i++;
        } else {
            R[k] = B[j];
            j++;
        }
}
```

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Merge(A[1..n_1], B[1..n_2], R[1..(n_1+n_2)]) {
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            R[k] = A[i];
            i++;
        } else {
            R[k] = B[j];
            j++;
        }
}
```

What most accurately characterizes the running time of Merge? ($n = n_1 + n_2$)

A. $O(1)$
B. $O(n)$
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D. None of the above

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Merge(A[1..n₁], B[1..n₂], R[1..(n₁+n₂)]) {
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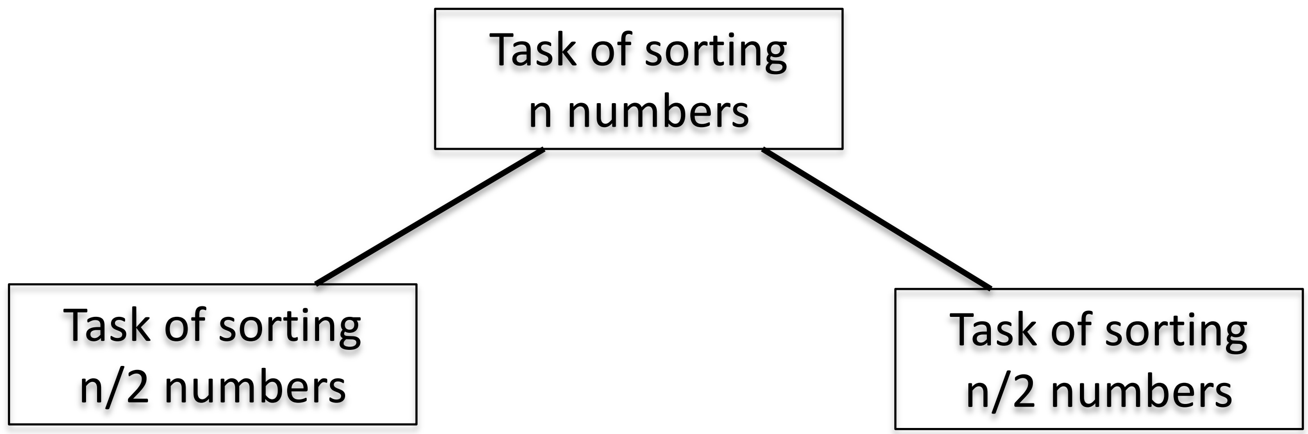
B. $O(n)$
C. $O(n^2)$
D. None of the above

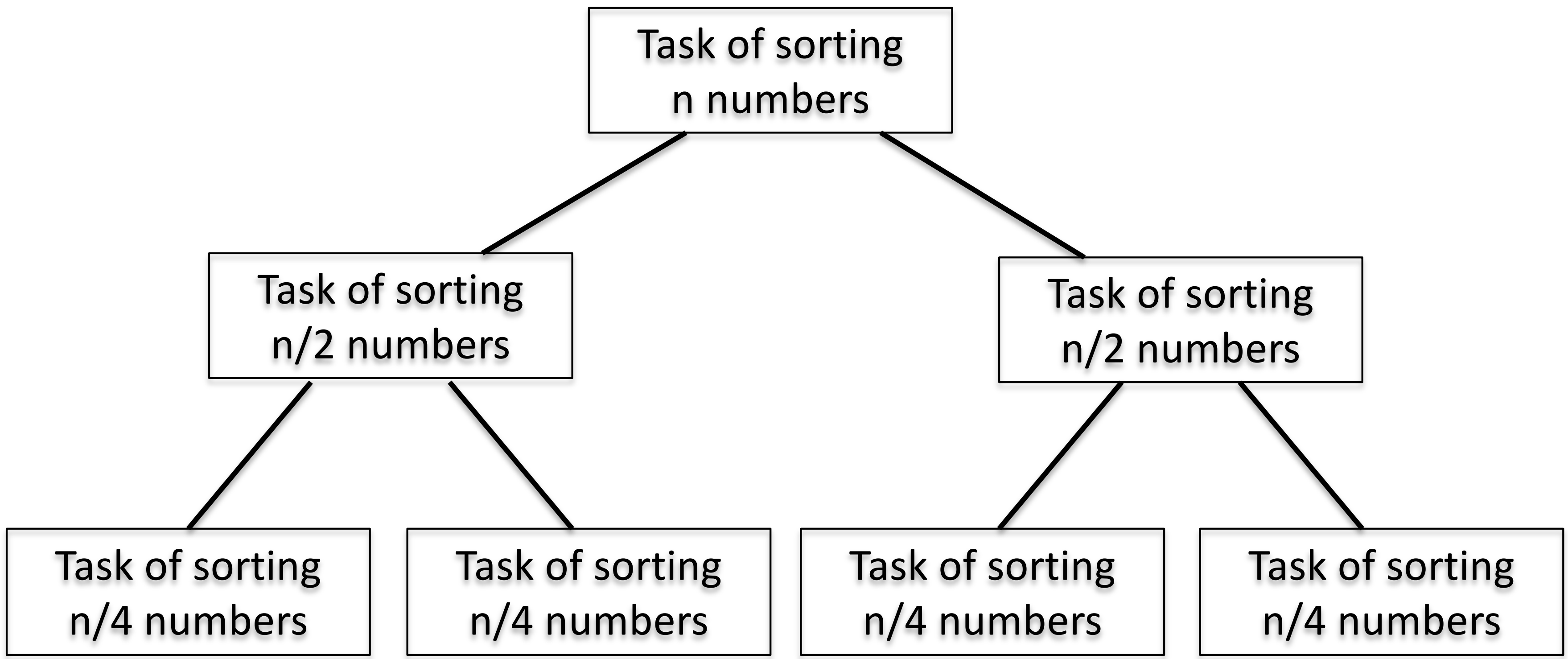
Running time of mergesort

Task of sorting
n numbers

Running time of mergesort

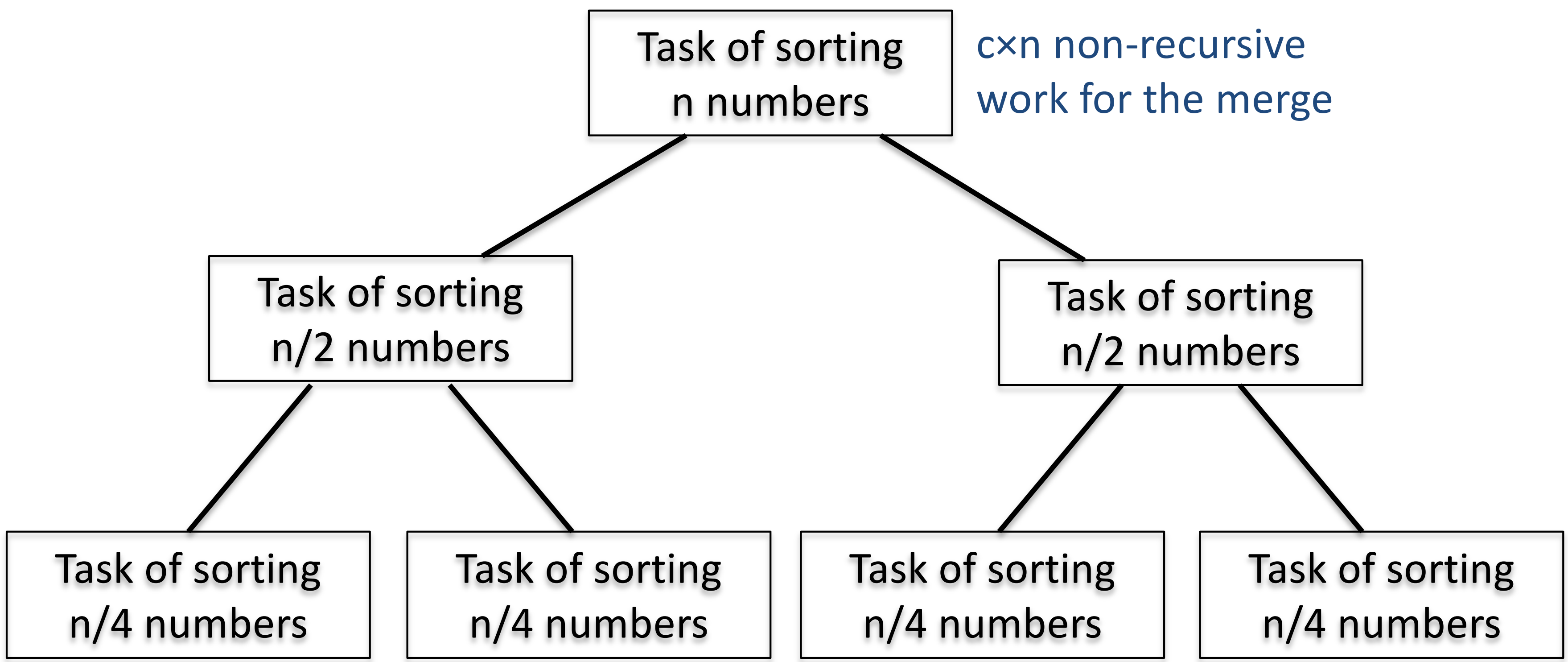


Running time of mergesort



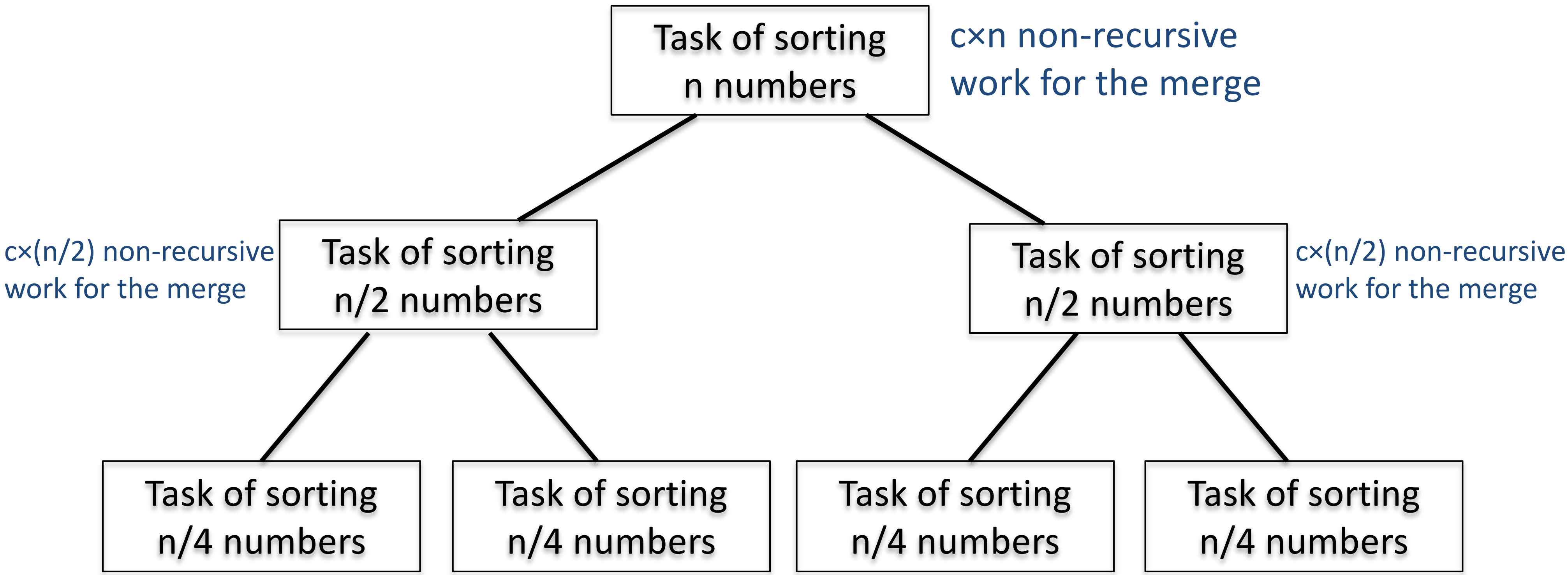
... and so on

Running time of mergesort



... and so on

Running time of mergesort



... and so on

Logarithms!

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$\log_b x$ = "log base b of x"

= power of b that gives x

= number of times you can divide x by b before getting 1

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= number of times you can divide x by b before getting 1

$2^0 = 1$ so $\log_2 1$ is 0

$2^1 = 2$ so $\log_2 2$ is 1

$2^2 = 4$ so $\log_2 4$ is 2 (and $\log_2 3$ is between 1 & 2)

What is $\log_2 32$?

A. 3
B. 4
C. 5
D. 6
E. None of the above

What is $\log_2 32$?

A. 3
B. 4
C. 5
D. 6
E. None of the above

What is $\log_2 100$?

A. Between 6 and 7
B. Between 7 and 8
C. Between 8 and 9
D. Between 9 and 10
E. None of the above

What is $\log_2 100$?

A. Between 6 and 7
B. Between 7 and 8
C. Between 8 and 9
D. Between 9 and 10
E. None of the above

What is $\log_2 1000$?

A. Between 7 and 8
B. Between 8 and 9
C. Between 9 and 10
D. Between 10 and 11
E. None of the above

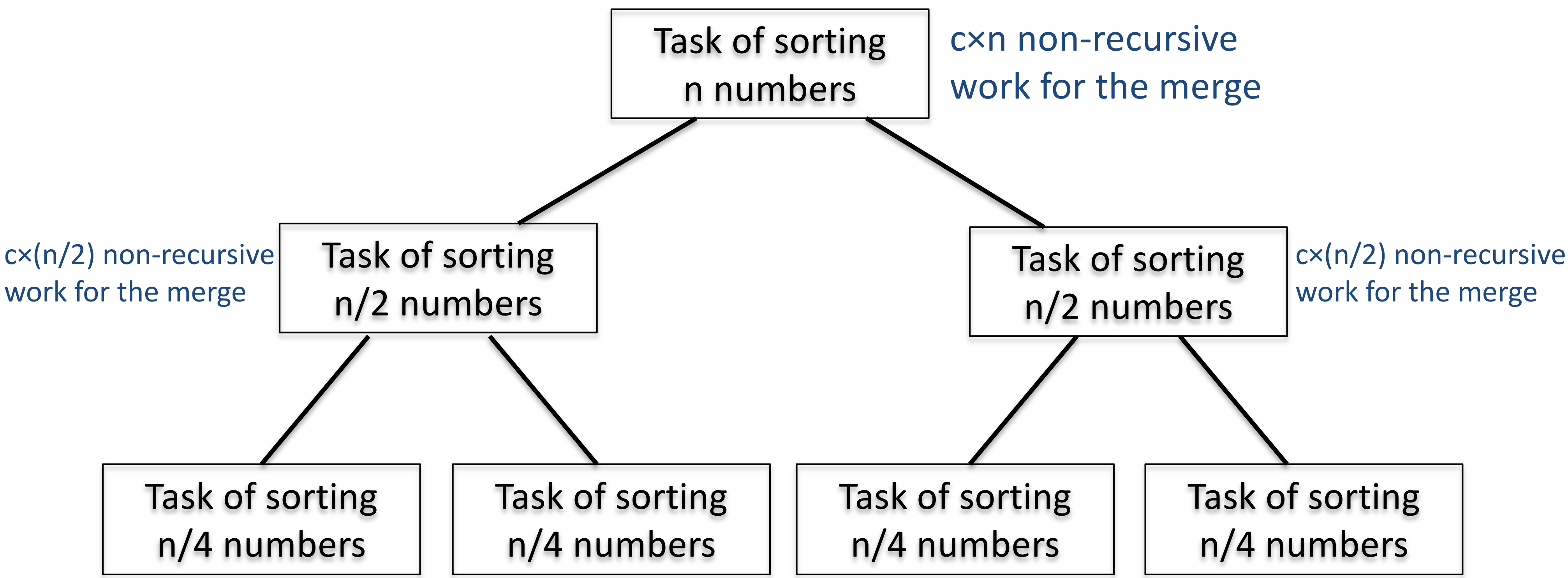
What is $\log_2 1000$?

A. Between 7 and 8
B. Between 8 and 9
C. Between 9 and 10
D. Between 10 and 11
E. None of the above

Properties of logarithms

- $\log_b xy = \log_b x + \log_b y$
- $\log_b (x/y) = \log_b x - \log_b y$
- $\log_b x^y = y \log_b x$
- $\log_b a = \dfrac{\log_c a}{\log_c b}$ for any c

Recall: Running time of mergesort



... and so on