CS 205: Algorithm design and analysis
Fall Term, 2024

Homework 4

Due: Tuesday 10/15 at 11:59pm

Complete the following. Submit all problems using Google Classroom.

1. (8 points) In class (9/30), we considered the problem of a consultant picking between high-stress and low-stress jobs. Suppose a different consultant reacts to stress differently and is able to work the week before a high-stress job, but cannot do high-stress jobs two weeks in a row. As before, there are n weeks, l_i is the payoff for doing a low-stress job during week i and h_i is the payoff for doing a high-stress job during week i. Give a dynamic programming algorithm to find the maximum payoff for this consultant.

2. (12 points) Recall the rod cutting problem from 9/27: You are given a rod of length n and an array A with $A[i]$ holding the value of a rod piece of length i. Your job is to find the way to cut the rod into pieces that maximizes its value. Unfortunately, it's near the end of the workday so your workers only have time for k cuts.

   (a) Give a dynamic programming algorithm that finds the maximum value pieces from a rod of length n that has been cut $\leq k$ times. (You're allowed to use fewer than k cuts since the workers won't object to going home early...) Hint: You'll need a two-dimensional table for this version of the problem.

   (b) Give a problem instance (value of n and table A) showing that the pieces created for this version of the problem might be entirely different sizes than for the version where you're allowed to use unlimited cuts. (Give the optimal solution for each case and observe that they have no sizes in common.)

3. (8 points) Use the matrix chain multiplication algorithm do decide how to multiply matrices $A_1A_2A_3A_4A_5$. The dimensions of these matrices are as follows: A_1 is 10×7, A_2 is 7×12, A_3 is 12×6, A_4 is 6×9, and A_5 is 9×11. Give the entire table for this problem's dynamic program (as discussed in class) and also the parenthesized list of matrices showing the order of multiplications that gives the smallest total number of operations.

4. (10 points) Each of the following can be solved using a graph algorithm we studied (BFS, DFS, shortest path/Dijkstra's, MST). For each problem, describe the graph (what are the vertices and edges?) and what algorithm you would use. Include a brief justification for your choice of graph algorithm.

   (a) The Knox website generates lots of information that is useful for admissions. It starts by identifying where vistors come from; did they do a web search to find Knox? (and what did they search for?) Did they come from a college ranking site? etc. The site also records the pages that each visitor goes to as they move from page to page. Using this information, you can create a representation of the website with a score for each pair of pages. A lower score means that visitor are more likely to select the second page when they visit the first one. The score for a sequence of pages is the sum of their pairwise scores; it indicates how likely a visitor is to traverse that entire sequence of pages and reach the last one. You have been asked to list all the pages from which it is possible to reach the Knox CS department homepage via a sequence with score at most S. How would you efficiently perform this task?

   (b) You are at a party and wondering about the relationships between people. Specifically, you have asked each person who else they know at the party (friends, roommates, relatives, classmates, etc). Now, you want to go through your records and generate a list of all the people connected to you: people who know you, people who know those people, etc. How would you generate such a list?

5. (1 point) Add a statement of collaboration to the top of your submission.