CS 205: Algorithm design and analysis
Fall Term, 2024

Homework 6

Due: Wednesday 11/6 at 11:59pm

Complete the following. Submit using Google Classroom in a file whose name is based on your name in that system.

1. (8 points) Before the course schedule can be finalized for a term, the Registrar's office must assign each class to a room. The inputs is a list of classes, each with the instructor and class period. (Assume each class meets for a single period; multi-period classes and clases at non-standard times are the bane of the registrar's existence.) In addition, each class comes with a list of rooms in which it can be scheduled based on class size, equipment needed, and instructor preference. Each class must be scheduled into a single room and no room is allowed to hold more than a single class during each period.

   Give an algorithm based on network flow to determine whether each class can be scheduled into one of the desired rooms during its period.

2. (5 points) When introducing algorithms based on network flow, I claimed that if all the edge capacities are integers, we could assume that the amount of flow sent along each edge of the maximum flow we found would be an integer. Explain why this is true for the Ford-Fulkerson algorithm.

3. (5 points) Give a network flow graph in which all edge capacities are integers and give a maximum flow on this graph where some edges carry an non-integer amount of flow. (The previous problem shows that a maximum flow with all integer amounts will also exist; your example just shows that not all maximum flows have this property.)

4. (8 points) In class, we talked about the problem of minimizing job flow time as a way to give users the best experience. This is not always the objective, though. Suppose you work for an unscrupulous company that wants to encourage users to pay for an OS upgrade. Thus, the company has decided to switch to a scheduling regimen that maximizes the average time users wait for their jobs to complete while never idling the processor (which would be too obvious). Specifically, you are given a set of n jobs, each with a duration p_i. Assuming all the jobs are available immediately (i.e. have release time 0), you want to maximize the total flow of these jobs. Show that Longest Job First, which runs jobs in order of non-increasing duration, achieves this goal.

5. (8 points) As a poor college student, you don't want to pay for video streaming services. Fortunately, these services have free trials, but unfortunately, each trial only lasts a short period. You plan on getting around this by working with your friends. Since each of you can sign up for your own free trial, a group of you can get multiple free trials as long as you watch the shows together.

   A group of you have put together a list of n dates when you want to have an active free trial, i.e. the days when your favorite shows will release new episodes. Each free trial lasts for 14 days. Give a greedy algorithm that finds the minimum number of free trials needed so that one will be active on each of your desired dates. Prove that it works.