1. (4 points) Your new job requires that you cross-reference data from two sorted arrays, each containing 
n distinct elements. Your first task is to find the median of the combined data set (i.e. the \( n^{th} \) smallest 
value in the pair of arrays). Give an algorithm to do this while looking at only \( O(\log n) \) data elements 
from either array. Be sure to justify your algorithm’s correctness and explain its running time.

2. (4 points) In an array \( A \), a fixed point is a value \( i \) such that \( A[i] = i \). Give an \( O(\log n) \)-time algorithm 
for finding the largest and smallest fixed points in a sorted array of distinct integers. Be sure to justify 
both the algorithm’s correctness and running time. (Hint: First, give an algorithm to find a fixed point 
in logarithmic time. Then, use the observation that all fixed points in an array of distinct integers are 
contiguous to show how to find the entire region of fixed points. Be careful to use only logarithmic time 
for this last step.)

3. (4 points) After graduation, you take a job as a fence builder. Your first assignment is to subdivide a 
square field holding sheep and cattle. Fences need to be built to separate the field into pieces so only 
one type of animal is in each piece. The rancher is willing to pay you to build one north-south fence 
across the entire field and one east-west fence for each of the resulting halves. For example, the field 
could be subdivided in the following way:

```
Cattle          Sheep

Sheep

Sheep          Cattle
```

Give a linear-time algorithm to determine if these fences can be built in such a way to separate the 
animals. Be sure to justify both the algorithm’s correctness and running time. The input is the animal 
positions (you’ll build the fences at night so none of the animals move). (Hint: The key is to find the 
position of the north-south fence. How can you determine if a possible position for it is correct?)