Problem Set 3

Due: Tuesday 1/27 at the beginning of class

Complete the following, which can be submitted via email or on paper.

1. (4 points)
   (a) Compute the height and balance for each of the nodes in the following AVL tree:

   ![AVL Tree Diagram]

   (b) Draw the tree after 16 is inserted and after any necessary rebalancing operations have occurred. Also describe the rebalancing operations, listing the specific rotation(s) performed.

   (c) Draw the tree after 18 is inserted (beginning with the tree from part (b)) and after any necessary rebalancing operations have occurred. Also describe the rebalancing operations, listing the specific rotation(s) performed.

2. (4 points) Consider a modification of the rod-cutting problem in which, in addition to a price \( p_i \) for each rod, each cut incurs a fixed cost \( c \). The profit associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. Give a dynamic programming algorithm to solve this modified problem.

3. (4 points) Knox recently acquired a meeting hall, which it hopes to rent out for different events. One complication with this plan is that there are two kinds of events: banquets and dances. Banquets require that tables be arranged to fill the room while dances require that the entire room be empty. Switching between these configurations incurs a cost \( s \). Fortunately, the events themselves generate revenue. Information on this comes from two arrays of \( n \) elements, \( B \) and \( D \). \( B[i] \) gives the amount for which the hall can be rented on night \( i \) in the banquet configuration and \( D[i] \) gives the corresponding amount for the dance configuration. Suppose that you are the event planner for the new facility. Your goal is to maximize the hall’s profit (the income it generates from rentals minus the cost for any switches that are needed). Assume that the hall is initially in the dance configuration.

   (a) A colleague proposes the simple algorithm of each night picking whichever configuration maximizes that night’s profit (i.e. the better of the current configuration’s rental income and the other configuration’s income minus the switching cost). Give an instance demonstrating that this strategy is not optimal.

   (b) Give an efficient algorithm to determine which events should be booked to maximize profit.