Problem Set 4

Due: Wednesday 2/11 at 4pm

Complete the following, which can be submitted via email or on paper.

1. (4 points) Complete Exercise 23.2-8 from the text (pages 637–638).

2. (4 points) When designing buildings, architects must be concerned with how easy the building is to evacuate in a fire or similar emergency. Your job is to help them evaluate a possible floorplan. The floorplan consists of rooms, pairs of which are connected by interior doors. Some rooms also have fire exits. The architects assume that they know everyone’s position (working at their desks of course) and that everyone knows exactly how to reach the closest fire exit by the most direct route. The architects further assume that it takes one time unit for a person can exit any room they are currently in, either by using one of its interior doors to enter another room or by using the fire exit to escape the building. This is possible no matter how other people are moving in the building (i.e. no one gets in anyone else’s way).

The safety requirement is that all the people must be able to exit the building in $k$ time units or less. How would you efficiently determine if a given building and set of people’s starting positions allows this goal to be achieved using a graph algorithm discussed in class? Describe how you would convert a problem instance into a graph. (What are the vertices, edges, and any weights assigned to edges?) Then describe how to solve the problem. Try to use an algorithm from class and describe any modifications needed in detail.


4. (4 points) In order to have cell phone service, a customer must be within distance $d$ of a cell tower. Your job is to determine how to place these towers to provide coverage along a country road. It is not necessary (i.e. too expensive) to provide continuous coverage so the cell phone company has decided to guarantee coverage only at “points of interest” along the road, such as rest stops and intersections. Assuming that the road is straight and you are given the location of these points of interest (represented as distances from the eastern end of the road), the problem is to determine the minimum number of cell towers needed in order for every point of interest to be within distance $d$ of one of them.

   (a) Give a counterexample showing that the idea of repeatedly placing a tower to cover the most houses doesn’t work.

   (b) Give an efficient optimal algorithm (and prove that it works).