Homework 2
Solution Key

Problem 2.1

IF item is cylindrical
THEN item is soda can (MB = \(\frac{2}{3}\))

IF item is soda can
AND item weighs under two ounces
THEN item is garbage (MB = \(\frac{5}{6}\))

IF item was in garbage area
THEN item is garbage (MB = \(\frac{9}{10}\))

IF item has sticker
THEN item is NOT garbage (MD = \(\frac{4}{5}\))

The robot trundles into an office and detects an item that she
is 99\% sure weighs one ounce, \(\frac{9}{10}\) certain is cylindrical, and \(\frac{8}{10}\) certain was in the garbage area. However, her vision system
suggests with low (\(\frac{1}{5}\)) certainty that it has a sticker on it.

Using the standard operations on Stanford certainty factors, de-
cide whether the robot thinks this item is garbage or not, and
give the certainty she has about this evaluation.

This problem is all about just working through the process, firing the ap-
propriate rules, and manipulating the numbers correctly.

We could fire the third rule first. We believe its antecedent to be true
with certainty \(\frac{8}{10}\), and the rule itself is believed with measure \(\frac{9}{10}\), so the
consequent “item is garbage” is believed with measure \(\frac{8}{10} \cdot \frac{9}{10} = \frac{72}{100}\). It
might seem like we could stop here, but it’s possible that other rule firings
will increase or decrease our certainty, so we have to keep going.

The first rule fires, and its antecedent has MB= \(\frac{9}{10}\), with an overall rule
certainty factor of \(\frac{2}{5}\), so we add to our belief collection that the item is a
soda can, in which fact our measure of belief is \(\frac{9}{10} \cdot \frac{2}{3} = \frac{3}{5}\).

This permits the second rule to fire. The antecedent is compound, with
the first part being believed with measure \(\frac{3}{5}\), and the second part with near
certainty (CF=.99). Since they are connected by AND, we take their min,
and so the consequent will be believed as a result with measure \(\frac{3}{5} \cdot \frac{5}{6} = \frac{1}{2}\).
This can be combined with our existing believe that the item is, in fact, garbage. Previously, we believed that with \( MB = \frac{72}{100} \), and now we corroborate it with \( MB = \frac{1}{2} \). Using the expression for combining concurring evidence, we now believe that the item is garbage with \( MB = \frac{72}{100} + \frac{1}{2} - \frac{72}{100} \cdot \frac{1}{2} \) or \( \frac{86}{100} \).

Then the final rule fires: the antecedent is believed with \( CF = \frac{1}{5} \), so we now have evidence against the item being garbage, and our MD as a result of this rule would be \( \frac{4}{25} \), meaning a CF of \( -\frac{4}{25} \) for the proposition that the item is garbage.

To combine this with the previous certainties, we use the expression for conflicting evidence:

\[
CF = \frac{\frac{86}{100} + -\frac{4}{25}}{1 - \min(\left| \frac{86}{100} \right|, \left| -\frac{4}{25} \right|)}
\]

\[
= \frac{\frac{70}{100}}{1 - \frac{4}{25}}
\]

\[
= \frac{70}{84} = \frac{5}{6}
\]

**Problem 2.2**

*IF* their grades are high  
*AND* they have good connections  
*THEN* their starting salary will be high  

*IF* their grades are high  
*AND* they have bad connections  
*THEN* their starting salary will be medium

*IF* their grades are low  
*AND* they have good connections  
*THEN* their starting salary will be high

*IF* their grades are low  
*AND* they have bad connections  
*THEN* their starting salary will be low

*On the grades scale* (measured by GPA):

\[
\text{high} = \{0/2.3, \ 1/3.8\}
\]

\[
\text{low} = \{1/2.3, \ 0/3.8\}
\]

*On the starting salary scale:*

\[
\text{lucrative} = \{0/60K, \ 1/100K\}
\]

\[
\text{high} = \{0/50K, \ 1/60K, \ 0/100K\}
\]

\[
\text{medium} = \{0/15K, \ 1/30K, \ 1/50K, \ 0/60K\}
\]

\[
\text{low} = \{1/15K, \ 0/30K\}
\]
...Draw graphs of the grades and salary scales. Then, consider a student with decent (good = .6, bad = .4) connections and a 3.5 GPA, and use fuzzy logic to estimate their starting salary.

The graph of our eventual target salary sets:

The graph of the grades sets (high and low) is as shown in this diagram (with the 3.5 GPA drawn in):

To calculate 3.5’s membership in the “high” and “low” sets, note that it is proportionally \( \frac{4}{5} \) of the distance from 2.3 to 3.8; it is thus .8 in the high set and .2 in the low set.

Meanwhile we’ve been given that the student’s connections are .6 good and .4 bad.

Since we’re using the max/min operations, for AND we take the minimum of two set memberships, and thus the four rules fire at .6, .4, .2, and .2 respectively. This means that our consequent claims are that the salary set will be the union of

- .6 high,
- .4 medium,
• .2 high, and
• .2 low.

Plotting these on the salary graph gives us

The union of these areas is our fuzzy output set, of which we must calculate the centroid (i.e. centre of gravity). Rather than compute it exactly, we can sample:

<table>
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<th>Salary</th>
<th>membership</th>
<th>Salary</th>
<th>membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>0K</td>
<td>.2</td>
<td>50K</td>
<td>.4</td>
</tr>
<tr>
<td>5K</td>
<td>.2</td>
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<td>.5</td>
</tr>
<tr>
<td>10K</td>
<td>.2</td>
<td>60K</td>
<td>.6</td>
</tr>
<tr>
<td>15K</td>
<td>.2</td>
<td>65K</td>
<td>.6</td>
</tr>
<tr>
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<td>.333</td>
<td>70K</td>
<td>.6</td>
</tr>
<tr>
<td>25K</td>
<td>.4</td>
<td>75K</td>
<td>.6</td>
</tr>
<tr>
<td>30K</td>
<td>.4</td>
<td>80K</td>
<td>.5</td>
</tr>
<tr>
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<td>.4</td>
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</tr>
<tr>
<td>40K</td>
<td>.4</td>
<td>90K</td>
<td>.25</td>
</tr>
<tr>
<td>45K</td>
<td>.4</td>
<td>95K</td>
<td>.125</td>
</tr>
</tbody>
</table>

Multiplying each salary by its membership and summing, we get a weighted sum of 398.4165; the sum of the weights (the membership values) is 7.6833, making the $x$-coordinate of the centroid, representing the expected salary of this student, $\$51,854.68$. 

4
Consider the following set of questions that can be asked about any given weekday in spring term:

- How many visitors are scheduled for that day? \( (V) \)
- Is the day in late April? \( (A) \)
- Did the caf receive a big shipment of cornbread mix in the preceding four-day period? \( (S) \)
- Is Helmut trying new recipes during that period? \( (C) \)
- Does the Farmer’s Almanac predict rain for that day? \( (R) \)
- Is the day Flunk Day? \( (F) \)

Arrange these questions into a Bayes net. For the purposes of this problem, take at face value the Union Board claim that they could schedule Flunk Day in any week of the term, on any day. Then,

a. Draw the Bayes net.

b. Without worrying about actual numbers, say how you would evaluate the probability of, say, Thursday the 26th of April being Flunk Day. That is, use symbolic probabilities like \( p(x|y) \) and so on.

(This answer is given in considerably more detail than I was requiring from you. A variety of Bayes nets are defensible here.)

This is how I suspect the various facts are interrelated:

```
      A
     / \  
    R   F
   /     /
  V     S
     \   /  
      C
```

This is Problem 2.3.
From there, I need to look at what I know. Since I’ve been given a specific date, my value for $A$ is known to be true, and since it’s close, I can look up in the almanac whether rain is predicted, so $R$ is also known (and since we’re symbolic I’ll assume ‘true’ for the discussion, but the reverse wouldn’t change the reasoning). I explicitly don’t know the value of $F$, and I probably am not going to know about Helmut, but $V$ and $S$ are theoretically knowable if I have moles in admissions and food services respectively.

I’m trying to estimate which is greater, $p(F = t|ARCSV)$ or $p(F = f|ARCSV)$. Because the conditioning environments are the same, this is equivalent to asking which joint probability is higher, $p(F = t, A, R, C, S, V)$ or $p(F = f, A, R, C, S, V)$. These in turn can be chained out into a bunch of conditional probabilities. For the first one:

$$p(A)p(R|A)p(F = t|R, A)p(V|F = t, R, A)p(C|V, F = t, R, A)p(S|C, V, F = t, R, A)$$

(and *mutatis mutandis* the same for the $F = f$ case, which I won’t write out explicitly). The order of these was chosen according to a topological sorting, so that I never added a descendant node within the DAG before adding its ancestor—this keeps my cause and effects straight, which is nice when you can do it. The independence assumptions encoded in the Bayes net let me simplify this to

$$p(A)p(R|A)p(F = t|R)p(V|F = t, A)p(C)p(S|C, F = t)$$

Now I can fill in the known details:

$$p(A = t)p(R = t|A = t)p(F = t|R = t)p(V|F = t, A = t)p(C)p(S|C, F = t)$$

and figure out what to do next. First of all, the first two terms are fully known but don’t involve $F$, the variable in question. We can effectively factor them out (just as we do the denominator) since they will affect everything equally. Leaving

$$p(F = t|R = t)p(V|F = t, A = t)p(C)p(S|C, F = t)$$

to compete with

$$p(F = f|R = t)p(V|F = f, A = t)p(C)p(S|C, F = f)$$

Formally, at this point, we would sum over all possible values of $C$, $V$, and $S$, so for the first one

$$\sum_{v \in V} \sum_{c \in C} \sum_{s \in S} p(F = t|R = t)p(V = v|F = t, A = t)p(C = c)p(S = s|C = c, F = t)$$
However, if $V$ is not known, all we’re doing with that summation is distributing the multiplication of several values that add to 1. Likewise, if $C$ and $S$ are both unknown, there is no point in summing. So you might just end up with

$$p(F = t|R = t)$$

If you knew $V$ to be, say, 3, but not the other stuff, you could say

$$p(F = t|R = t)p(V = 3|F = t, A = t)$$

If you didn’t know $C$ but did know $S$, you can’t simply eliminate the summation over $C$, because the final probability has the probability of a given value of $S$ based on the various values of $C$. So if you knew that there had been a shipment of cornbread mix (thus $S = t$), you would evaluate

$$\sum_{c \in C} p(F = t|R = t)p(C = c)p(S = t|C = c, F = t)$$

(again, along with its $F = f$ counterpart).

That’s about as far as you can go on this problem without numbers to work with....