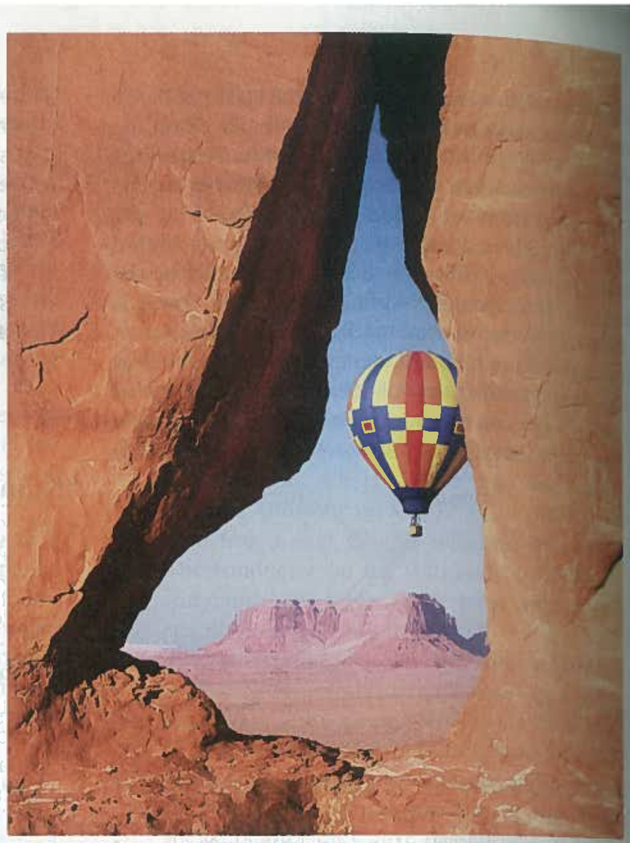
Heating the air inside a "hot-air" balloon raises the air's temperature, causing it to expand, and forces air out the opening at the bottom. The reduced amount of air inside means its density is lower than the outside air, so there is a net buoyant force upward on the balloon. In this Chapter we study temperature and its effects on matter: thermal expansion and the gas laws.



CHAPTER 17

Temperature, Thermal Expansion, and the Ideal Gas Law

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

A hot-air balloon, open at one end (see photos above), rises when the air inside is heated by a flame. For the following properties, is the air inside the balloon higher, lower, or the same as for the air outside the balloon?

(i) Temperature,
(ii) Pressure,
(iii) Density.

In the next four Chapters, Chapters 17 through 20, we study temperature, heat and thermodynamics, and the kinetic theory of gases.

We will often consider a particular **system**, by which we mean a particular object or set of objects; everything else in the universe is called the "environment." We can describe the **state** (or condition) of a particular system—such as a gas in a container—from either a microscopic or macroscopic point of view. A **microscopic** description would involve details of the motion of all the atoms or molecules making up the system, which could be very complicated. A **macroscopic** description is given in terms of quantities that are detectable directly by our senses and instruments, such as volume, mass, pressure, and temperature.

The description of processes in terms of macroscopic quantities is the field of **thermodynamics**. Quantities that can be used to describe the state of a system are called **state variables**. To describe the state of a pure gas in a container, for example, requires only three state variables, which are typically the volume, the pressure, and the temperature. More complex systems require more than three state variables to describe them.

The emphasis in this Chapter is on the concept of temperature. We begin, however, with a brief discussion of the theory that matter is made up of atoms and that these atoms are in continual random motion. This theory is called *kinetic theory* ("kinetic," you may recall, is Greek for "moving"), and we discuss it in more detail in Chapter 18.

17–1 Atomic Theory of Matter

The idea that matter is made up of atoms dates back to the ancient Greeks. According to the Greek philosopher Democritus, if a pure substance—say, a piece of iron—were cut into smaller and smaller bits, eventually a smallest piece of that substance would be obtained which could not be divided further. This smallest piece was called an **atom**, which in Greek means "indivisible."[†]

Today the atomic theory is universally accepted. The experimental evidence in its favor, however, came mainly in the eighteenth, nineteenth, and twentieth centuries, and much of it was obtained from the analysis of chemical reactions.

We will often speak of the relative masses of individual atoms and molecules—what we call the **atomic mass** or **molecular mass**, respectively.[‡] These are based on arbitrarily assigning the abundant carbon atom, ^{12}C, the atomic mass of exactly 12.0000 **unified atomic mass units** (u). In terms of kilograms,

$$1\,\text{u} = 1.6605 \times 10^{-27}\,\text{kg}.$$

The atomic mass of hydrogen is then 1.0078 u, and the values for other atoms are as listed in the Periodic Table inside the back cover of this book, and also in Appendix F. The molecular mass of a compound is the sum of atomic masses of the atoms making up the molecules of that compound.[§]

An important piece of evidence for the atomic theory is called **Brownian motion**, named after the biologist Robert Brown, who is credited with its discovery in 1827. While he was observing tiny pollen grains suspended in water under his microscope, Brown noticed that the tiny grains moved about in tortuous paths (Fig. 17–1), even though the water appeared to be perfectly still. The atomic theory easily explains Brownian motion if the further reasonable assumption is made that the atoms of any substance are continually in motion. Then Brown's tiny pollen grains are jostled about by the vigorous barrage of rapidly moving molecules of water.

In 1905, Albert Einstein examined Brownian motion from a theoretical point of view and was able to calculate from the experimental data the approximate size and mass of atoms and molecules. His calculations showed that the diameter of a typical atom is about 10^{-10} m.

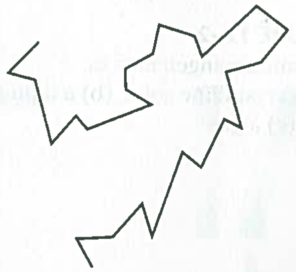


FIGURE 17–1 Path of a tiny particle (pollen grain, for example) suspended in water. The straight lines connect observed positions of the particle at equal time intervals.

[†] Today we do not consider the atom as indivisible, but rather as consisting of a nucleus (containing protons and neutrons) and electrons.

[‡] The terms *atomic weight* and *molecular weight* are sometimes used for these quantities, but properly speaking we are comparing masses.

[§] An *element* is a substance, such as gold, iron, or copper, that cannot be broken down into simpler substances by chemical means. *Compounds* are substances made up of elements, and can be broken down into them; examples are carbon dioxide and water. The smallest piece of an element is an atom; the smallest piece of a compound is a molecule. Molecules are made up of atoms; a molecule of water, for example, is made up of two atoms of hydrogen and one of oxygen; its chemical formula is H_2O.

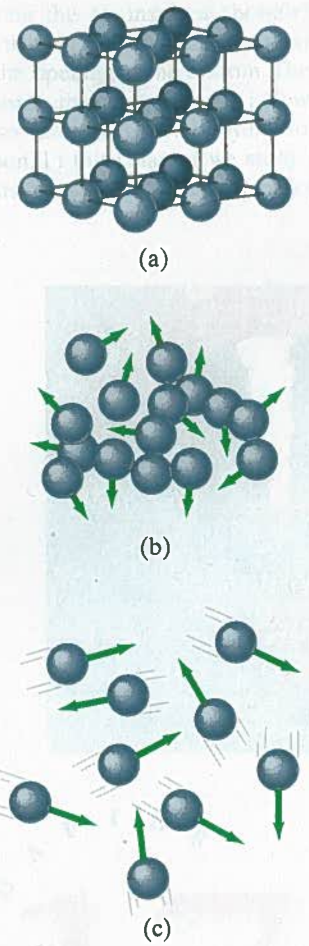


FIGURE 17–2 Atomic arrangements in (a) a crystalline solid, (b) a liquid, and (c) a gas.

FIGURE 17–3 Expansion joint on a bridge.

At the start of Chapter 13, we distinguished the three common phases (or states) of matter—solid, liquid, gas—based on **macroscopic**, or "large-scale," properties. Now let us see how these three phases of matter differ, from the atomic or **microscopic** point of view. Clearly, atoms and molecules must exert attractive forces on each other. For how else could a brick or a block of aluminum hold together in one piece? The attractive forces between molecules are of an electrical nature (more on this in later Chapters). When molecules come too close together, the force between them must become repulsive (electric repulsion between their outer electrons), for how else could matter take up space? Thus molecules maintain a minimum distance from each other. In a solid material, the attractive forces are strong enough that the atoms or molecules move only slightly (oscillate) about relatively fixed positions, often in an array known as a crystal lattice, as shown in Fig. 17–2a. In a liquid, the atoms or molecules are moving more rapidly, or the forces between them are weaker, so that they are sufficiently free to pass around one another, as in Fig. 17–2b. In a gas, the forces are so weak, or the speeds so high, that the molecules do not even stay close together. They move rapidly every which way, Fig. 17–2c, filling any container and occasionally colliding with one another. On average, the speeds are sufficiently high in a gas that when two molecules collide, the force of attraction is not strong enough to keep them close together and they fly off in new directions.

EXAMPLE 17–1 **ESTIMATE** **Distance between atoms.** The density of copper is $8.9 \times 10^3\ \mathrm{kg/m^3}$, and each copper atom has a mass of 63 u. Estimate the average distance between the centers of neighboring copper atoms.

APPROACH We consider a cube of copper 1 m on a side. From the given density ρ we can calculate the mass m of a cube of volume $V = 1\ \mathrm{m^3}$ $(m = \rho V)$. We divide this by the mass of one atom (63 u) to obtain the number of atoms in $1\ \mathrm{m^3}$. We assume the atoms are in a uniform array, and we let N be the number of atoms in a 1-m length; then $(N)(N)(N) = N^3$ equals the total number of atoms in $1\ \mathrm{m^3}$.

SOLUTION The mass of 1 copper atom is $63\ \mathrm{u} = 63 \times 1.66 \times 10^{-27}\ \mathrm{kg} = 1.05 \times 10^{-25}\ \mathrm{kg}$. This means that in a cube of copper 1 m on a side $(\text{volume} = 1\ \mathrm{m^3})$, there are

$$\frac{8.9 \times 10^3\ \mathrm{kg/m^3}}{1.05 \times 10^{-25}\ \mathrm{kg/atom}} = 8.5 \times 10^{28}\ \mathrm{atoms/m^3}.$$

The volume of a cube of side ℓ is $V = \ell^3$, so on one edge of the 1-m-long cube there are $(8.5 \times 10^{28})^{\frac{1}{3}}$ atoms $= 4.4 \times 10^9$ atoms. Hence the distance between neighboring atoms is

$$\frac{1\ \mathrm{m}}{4.4 \times 10^9\ \mathrm{atoms}} = 2.3 \times 10^{-10}\ \mathrm{m}.$$

NOTE Watch out for units. Even though "atoms" is not a unit, it is helpful to include it to make sure you calculate correctly.

17–2 Temperature and Thermometers

In everyday life, **temperature** is a measure of how hot or cold something is. A hot oven is said to have a high temperature, whereas the ice of a frozen lake is said to have a low temperature.

Many properties of matter change with temperature. For example, most materials expand when their temperature is increased.[†] An iron beam is longer when hot than when cold. Concrete roads and sidewalks expand and contract slightly according to temperature, which is why compressible spacers or expansion joints (Fig. 17–3) are placed at regular intervals. The electrical resistance of matter changes with temperature (Chapter 25). So too does the color radiated by objects, at least at high temperatures: you may have noticed that the heating element of an electric stove glows with a red color when hot. At higher temperatures, solids such as iron glow orange or even white. The white light from an ordinary incandescent lightbulb comes from an extremely hot tungsten wire. The surface temperatures of the Sun and other stars can be measured by the predominant color (more precisely, wavelengths) of light they emit.

Instruments designed to measure temperature are called **thermometers**. There are many kinds of thermometers, but their operation always depends on some property of matter that changes with temperature. Many common thermometers rely on the expansion of a material with an increase in temperature. The first idea for a thermometer, by Galileo, made use of the expansion of a gas. Common thermometers today consist of a hollow glass tube filled with mercury or with alcohol colored with a red dye, as were the earliest usable thermometers (Fig. 17–4).

Inside a common liquid-in-glass thermometer, the liquid expands more than the glass when the temperature is increased, so the liquid level rises in the tube (Fig. 17–5a). Although metals also expand with temperature, the change in length of a metal rod, say, is generally too small to measure accurately for ordinary changes in temperature. However, a useful thermometer can be made by bonding together two dissimilar metals whose rates of expansion are different (Fig. 17–5b). When the temperature is increased, the different amounts of expansion cause the bimetallic strip to bend. Often the bimetallic strip is in the form of a coil, one end of which is fixed while the other is attached to a pointer, Fig. 17–6. This kind of thermometer is used as ordinary air thermometers, oven thermometers, automatic off switches in electric coffeepots, and in room thermostats for determining when the heater or air conditioner should go on or off. Very precise thermometers make use of electrical properties (Chapter 25), such as resistance thermometers, thermocouples, and thermistors, often with a digital readout.

Temperature Scales

In order to measure temperature quantitatively, some sort of numerical scale must be defined. The most common scale today is the **Celsius** scale, sometimes called the **centigrade** scale. In the United States, the **Fahrenheit** scale is also common. The most important scale in scientific work is the absolute, or Kelvin, scale, and it will be discussed later in this Chapter.

One way to define a temperature scale is to assign arbitrary values to two readily reproducible temperatures. For both the Celsius and Fahrenheit scales these two fixed points are chosen to be the freezing point and the boiling point[†] of water, both taken at standard atmospheric pressure. On the Celsius scale, the freezing point of water is chosen to be 0°C ("zero degrees Celsius") and the boiling point 100°C. On the Fahrenheit scale, the freezing point is defined as 32°F and the boiling point 212°F. A practical thermometer is calibrated by placing it in carefully prepared environments at each of the two temperatures and marking the position of the liquid or pointer. For a Celsius scale, the distance between the two marks is divided into one hundred equal intervals representing each degree between 0°C and 100°C (hence the name "centigrade scale" meaning "hundred steps"). For a Fahrenheit scale, the two points are labeled 32°F and 212°F and the distance between them is divided into 180 equal intervals. For temperatures below the freezing point of water and above the boiling point of water, the scales may be extended using the same equally spaced intervals. However, thermometers can be used only over a limited temperature range because of their own limitations—for example, the liquid mercury in a mercury-in-glass thermometer solidifies at some point, below which the thermometer will be useless. It is also rendered useless above temperatures where the fluid, such as alcohol, vaporizes. For very low or very high temperatures, specialized thermometers are required, some of which we will mention later.

FIGURE 17–4 Thermometers built by the Accademia del Cimento (1657–1667) in Florence, Italy, are among the earliest known. These sensitive and exquisite instruments contained alcohol, sometimes colored, like many thermometers today.

FIGURE 17–5 (a) Mercury- or alcohol-in-glass thermometer; (b) bimetallic strip.

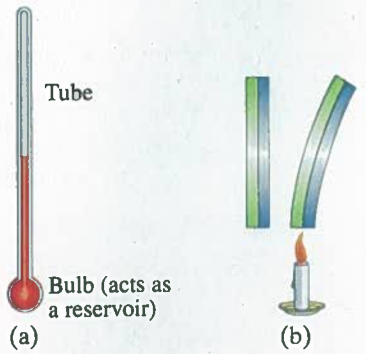
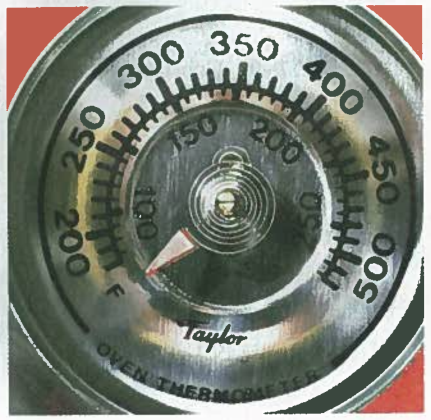


FIGURE 17–6 Photograph of a thermometer using a coiled bimetallic strip.



[†] Most materials expand when their temperature is raised, but not all. Water, for example, in the range 0°C to 4°C contracts with an increase in temperature (see Section 17–4).

[†] The freezing point of a substance is defined as that temperature at which the solid and liquid phases coexist in equilibrium—that is, without any net liquid changing into the solid or vice versa. Experimentally, this is found to occur at only one definite temperature, for a given pressure. Similarly, the boiling point is defined as that temperature at which the liquid and gas coexist in equilibrium. Since these points vary with pressure, the pressure must be specified (usually it is 1 atm).

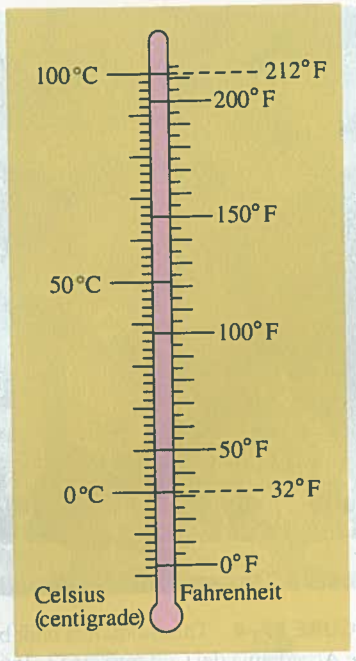


FIGURE 17–7 Celsius and Fahrenheit scales compared.

CAUTION

Convert temperature by remembering 0°C = 32°F and a change of 5 C° = 9 F°

Every temperature on the Celsius scale corresponds to a particular temperature on the Fahrenheit scale, Fig. 17–7. It is easy to convert from one to the other if you remember that 0°C corresponds to 32°F and that a range of 100° on the Celsius scale corresponds to a range of 180° on the Fahrenheit scale. Thus, one Fahrenheit degree (1 F°) corresponds to $100/180 = \frac{5}{9}$ of a Celsius degree (1 C°). That is, $1\text{ F}° = \frac{5}{9}\text{ C}°$. (Notice that when we refer to a specific temperature, we say "degrees Celsius," as in 20°C; but when we refer to a *change* in temperature or a temperature interval, we say "Celsius degrees," as in "2 C°.") The conversion between the two temperature scales can be written

$$T(°\text{C}) = \tfrac{5}{9}[T(°\text{F}) - 32]$$

or

$$T(°\text{F}) = \tfrac{9}{5}T(°\text{C}) + 32.$$

Rather than memorizing these relations (it would be easy to confuse them), it is usually easier simply to remember that 0°C = 32°F and that a change of 5 C° = a change of 9 F°.

EXAMPLE 17–2 **Taking your temperature.** Normal body temperature is 98.6°F. What is this on the Celsius scale?

APPROACH We recall that 0°C = 32°F and 5 C° = 9 F°.

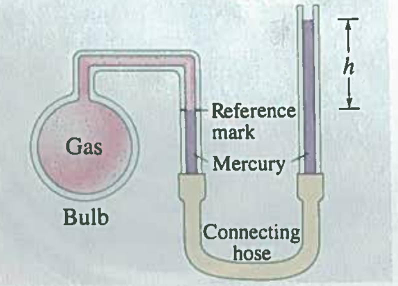
SOLUTION First we relate the given temperature to the freezing point of water (0°C). That is, 98.6°F is $98.6 - 32.0 = 66.6\text{ F}°$ above the freezing point of water. Since each F° is equal to $\frac{5}{9}$ C°, this corresponds to $66.6 \times \frac{5}{9} = 37.0$ Celsius degrees above the freezing point. The freezing point is 0°C, so the temperature is 37.0°C.

EXERCISE A Determine the temperature at which both scales give the same numerical reading $(T_\text{C} = T_\text{F})$.

Different materials do not expand in quite the same way over a wide temperature range. Consequently, if we calibrate different kinds of thermometers exactly as described above, they will not usually agree precisely. Because of how we calibrate them, they will agree at 0°C and at 100°C. But because of different expansion properties, they may not agree precisely at intermediate temperatures (remember we arbitrarily divided the thermometer scale into 100 equal divisions between 0°C and 100°C). Thus a carefully calibrated mercury-in-glass thermometer might register 52.0°C, whereas a carefully calibrated thermometer of another type might read 52.6°C. Discrepancies below 0°C and above 100°C can also be significant.

Because of such discrepancies, some standard kind of thermometer must be chosen so that all temperatures can be precisely defined. The chosen standard for this purpose is the **constant-volume gas thermometer.** As shown in the simplified diagram of Fig. 17–8, this thermometer consists of a bulb filled with a dilute gas connected by a thin tube to a mercury manometer (Section 13–6). The volume of the gas is kept constant by raising or lowering the right-hand tube of the manometer so that the mercury in the left-hand tube coincides with the reference mark. An increase in temperature causes a proportional increase in pressure in the bulb. Thus the tube must be lifted higher to keep the gas volume constant. The height of the mercury in the right-hand column is then a measure of the temperature. This thermometer gives the same results for all gases in the limit of reducing the gas pressure in the bulb toward zero. The resulting scale serves as a basis for the standard temperature scale (Section 17–10).

FIGURE 17–8 Constant-volume gas thermometer.



17–3 Thermal Equilibrium and the Zeroth Law of Thermodynamics

We are all familiar with the fact that if two objects at different temperatures are placed in thermal contact (meaning thermal energy can transfer from one to the other), the two objects will eventually reach the same temperature. They are then said to be in **thermal equilibrium**. For example, you leave a fever thermometer in your mouth until it comes into thermal equilibrium with that environment, and then you read it. Two objects are defined to be in thermal equilibrium if, when placed in thermal contact, no net energy flows from one to the other, and their temperatures don't change. Experiments indicate that

if two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.

This postulate is called the **zeroth law of thermodynamics**. It has this unusual name because it was not until after the great first and second laws of thermodynamics (Chapters 19 and 20) were worked out that scientists realized that this apparently obvious postulate needed to be stated first.

Temperature is a property of a system that determines whether the system will be in thermal equilibrium with other systems. When two systems are in thermal equilibrium, their temperatures are, by definition, equal, and no net thermal energy will be exchanged between them. This is consistent with our everyday notion of temperature, since when a hot object and a cold one are put into contact, they eventually come to the same temperature. Thus the importance of the zeroth law is that it allows a useful definition of temperature.

17–4 Thermal Expansion

Most substances expand when heated and contract when cooled. However, the amount of expansion or contraction varies, depending on the material.

Linear Expansion

Experiments indicate that the change in length $\Delta\ell$ of almost all solids is, to a good approximation, directly proportional to the change in temperature ΔT, as long as ΔT is not too large. The change in length is also proportional to the original length of the object, ℓ_0. That is, for the same temperature increase, a 4-m-long iron rod will increase in length twice as much as a 2-m-long iron rod. We can write this proportionality as an equation:

$$\Delta\ell = \alpha\ell_0\,\Delta T, \tag{17–1a}$$

where α, the proportionality constant, is called the *coefficient of linear expansion* for the particular material and has units of $(\text{C}°)^{-1}$. We write $\ell = \ell_0 + \Delta\ell$, Fig. 17–9, and rewrite this equation as $\ell = \ell_0 + \Delta\ell = \ell_0 + \alpha\ell_0\,\Delta T$, or

$$\ell = \ell_0(1 + \alpha\,\Delta T), \tag{17–1b}$$

where ℓ_0 is the length initially, at temperature T_0, and ℓ is the length after heating or cooling to a temperature T. If the temperature change $\Delta T = T - T_0$ is negative, then $\Delta\ell = \ell - \ell_0$ is also negative; the length shortens as the temperature decreases.

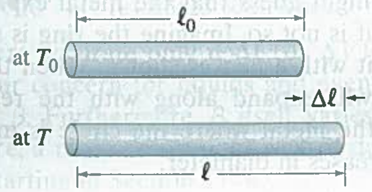


FIGURE 17–9 A thin rod of length ℓ_0 at temperature T_0 is heated to a new uniform temperature T and acquires length ℓ, where $\ell = \ell_0 + \Delta\ell$.

TABLE 17–1 Coefficients of Expansion, near 20°C

Material	Coefficient of Linear Expansion, α $(\text{C}^\circ)^{-1}$	Coefficient of Volume Expansion, β $(\text{C}^\circ)^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$
Liquids		
Gasoline		950×10^{-6}
Mercury		180×10^{-6}
Ethyl alcohol		1100×10^{-6}
Glycerin		500×10^{-6}
Water		210×10^{-6}
Gases		
Air (and most other gases at atmospheric pressure)		3400×10^{-6}

The values of α for various materials at 20°C are listed in Table 17–1. Actually, α does vary slightly with temperature (which is why thermometers made of different materials do not agree precisely). However, if the temperature range is not too great, the variation can usually be ignored.

PHYSICS APPLIED
Expansion in structures

EXAMPLE 17–3 **Bridge expansion.** The steel bed of a suspension bridge is 200 m long at 20°C. If the extremes of temperature to which it might be exposed are −30°C to +40°C, how much will it contract and expand?

APPROACH We assume the bridge bed will expand and contract linearly with temperature, as given by Eq. 17–1a.

SOLUTION From Table 17–1, we find that $\alpha = 12 \times 10^{-6}(\text{C}^\circ)^{-1}$ for steel. The increase in length when it is at 40°C will be

$$\Delta \ell = \alpha \ell_0 \Delta T = (12 \times 10^{-6}/\text{C}^\circ)(200\,\text{m})(40^\circ\text{C} - 20^\circ\text{C}) = 4.8 \times 10^{-2}\,\text{m},$$

or 4.8 cm. When the temperature decreases to −30°C, $\Delta T = -50\,\text{C}^\circ$. Then

$$\Delta \ell = (12 \times 10^{-6}/\text{C}^\circ)(200\,\text{m})(-50\,\text{C}^\circ) = -12.0 \times 10^{-2}\,\text{m},$$

or a decrease in length of 12 cm. The total range the expansion joints must accommodate is 12 cm + 4.8 cm ≈ 17 cm (Fig. 17–3).

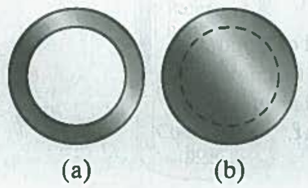


FIGURE 17–10 Example 17–4.

CONCEPTUAL EXAMPLE 17–4 **Do holes expand or contract?** If you heat a thin, circular ring (Fig. 17–10a) in the oven, does the ring's hole get larger or smaller?

RESPONSE You might guess that the metal expands into the hole, making the hole smaller. But it is not so. Imagine the ring is solid, like a coin (Fig. 17–10b). Draw a circle on it with a pen as shown. When the metal expands, the material inside the circle will expand along with the rest of the metal; so the circle expands. Cutting the metal where the circle is makes clear to us that the hole in Fig. 17–10a increases in diameter.

EXAMPLE 17–5 **Ring on a rod.** An iron ring is to fit snugly on a cylindrical iron rod. At 20°C, the diameter of the rod is 6.445 cm and the inside diameter of the ring is 6.420 cm. To slip over the rod, the ring must be slightly larger than the rod diameter by about 0.008 cm. To what temperature must the ring be brought if its hole is to be large enough so it will slip over the rod?

APPROACH The hole in the ring must be increased from a diameter of 6.420 cm to 6.445 cm + 0.008 cm = 6.453 cm. The ring must be heated since the hole diameter will increase linearly with temperature (Example 17–4).

SOLUTION We solve for ΔT in Eq. 17–1a and find

$$\Delta T = \frac{\Delta \ell}{\alpha \ell_0} = \frac{6.453\,\text{cm} - 6.420\,\text{cm}}{(12 \times 10^{-6}/\text{C}^\circ)(6.420\,\text{cm})} = 430\,\text{C}^\circ.$$

So it must be raised at least to $T = (20^\circ\text{C} + 430\,\text{C}^\circ) = 450^\circ\text{C}$.

NOTE In doing Problems, do not forget the last step, adding in the initial temperature (20°C here).

PHYSICS APPLIED
Opening a tight lid

CONCEPTUAL EXAMPLE 17–6 **Opening a tight jar lid.** When the lid of a glass jar is tight, holding the lid under hot water for a short time will often make it easier to open (Fig. 17–11). Why?

RESPONSE The lid may be struck by the hot water more directly than the glass and so expand sooner. But even if not, metals generally expand more than glass for the same temperature change (α is greater—see Table 17–1).

NOTE If you put a hard-boiled egg in cold water immediately after cooking it, it is easier to peel: the different thermal expansions of the shell and egg cause the egg to separate from the shell.

PHYSICS APPLIED
Peeling a hard-boiled egg

FIGURE 17–11 Example 17–6.

Volume Expansion

The change in *volume* of a material which undergoes a temperature change is given by a relation similar to Eq. 17–1a, namely,

$$\Delta V = \beta V_0 \Delta T, \tag{17–2}$$

where ΔT is the change in temperature, V_0 is the original volume, ΔV is the change in volume, and β is the *coefficient of volume expansion*. The units of β are $(\text{C}^\circ)^{-1}$.

Values of β for various materials are given in Table 17–1. Notice that for solids, β is normally equal to approximately 3α. To see why, consider a rectangular solid of length ℓ_0, width W_0, and height H_0. When its temperature is changed by ΔT, its volume changes from $V_0 = \ell_0 W_0 H_0$ to

$$V = \ell_0(1 + \alpha \Delta T) W_0(1 + \alpha \Delta T) H_0(1 + \alpha \Delta T),$$

using Eq. 17–1b and assuming α is the same in all directions. Thus,

$$\Delta V = V - V_0 = V_0(1 + \alpha \Delta T)^3 - V_0 = V_0\left[3\alpha \Delta T + 3(\alpha \Delta T)^2 + (\alpha \Delta T)^3\right].$$

If the amount of expansion is much smaller than the original size of the object, then $\alpha \Delta T \ll 1$ and we can ignore all but the first term and obtain

$$\Delta V \approx (3\alpha) V_0 \Delta T.$$

This is Eq. 17–2 with $\beta \approx 3\alpha$. For solids that are not isotropic (having the same properties in all directions), however, the relation $\beta \approx 3\alpha$ is not valid. Note also that linear expansion has no meaning for liquids and gases since they do not have fixed shapes.

EXERCISE B A long thin bar of aluminum at 0°C is 1.0 m long and has a volume of $1.0000 \times 10^{-3}\,\text{m}^3$. When heated to 100°C, the length of the bar becomes 1.0025 m. What is the approximate volume of the bar at 100°C? (a) $1.0000 \times 10^{-3}\,\text{m}^3$; (b) $1.0025 \times 10^{-3}\,\text{m}^3$; (c) $1.0050 \times 10^{-3}\,\text{m}^3$; (d) $1.0075 \times 10^{-3}\,\text{m}^3$; (e) $2.5625 \times 10^{-3}\,\text{m}^3$.

Equations 17–1 and 17–2 are accurate only if $\Delta \ell$ (or ΔV) is small compared to ℓ_0 (or V_0). This is of particular concern for liquids and even more so for gases because of the large values of β. Furthermore, β itself varies substantially with temperature for gases. Therefore, a more convenient way of dealing with gases is needed, and will be discussed starting in Section 17–6.

EXAMPLE 17–7 **Gas tank in the Sun.** The 70-liter (L) steel gas tank of a car is filled to the top with gasoline at 20°C. The car sits in the Sun and the tank reaches a temperature of 40°C (104°F). How much gasoline do you expect to overflow from the tank?

PHYSICS APPLIED
Gas tank overflow

APPROACH Both the gasoline and the tank expand as the temperature increases, and we assume they do so linearly as described by Eq. 17–2. The volume of overflowing gasoline equals the volume increase of the gasoline minus the increase in volume of the tank.

SOLUTION The gasoline expands by

$$\Delta V = \beta V_0 \Delta T = (950 \times 10^{-6}/\mathrm{C}^\circ)(70\,\mathrm{L})(40^\circ\mathrm{C} - 20^\circ\mathrm{C}) = 1.3\,\mathrm{L}.$$

The tank also expands. We can think of it as a steel shell that undergoes volume expansion $(\beta \approx 3\alpha = 36 \times 10^{-6}/\mathrm{C}^\circ)$. If the tank were solid, the surface layer (the shell) would expand just the same. Thus the tank increases in volume by

$$\Delta V = (36 \times 10^{-6}/\mathrm{C}^\circ)(70\,\mathrm{L})(40^\circ\mathrm{C} - 20^\circ\mathrm{C}) = 0.050\,\mathrm{L},$$

so the tank expansion has little effect. More than a liter of gas could spill out.

NOTE Want to save a few pennies? You pay for gas by volume, so fill your gas tank when it is cool and the gas is denser—more molecules for the same price. But don't fill the tank quite all the way.

Anomalous Behavior of Water Below 4°C

Most substances expand more or less uniformly with an increase in temperature, as long as no phase change occurs. Water, however, does not follow the usual pattern. If water at 0°C is heated, it actually *decreases* in volume until it reaches 4°C. Above 4°C water behaves normally and expands in volume as the temperature is increased, Fig. 17–12. Water thus has its greatest density at 4°C. This anomalous behavior of water is of great importance for the survival of aquatic life during cold winters. When the water in a lake or river is above 4°C and begins to cool by contact with cold air, the water at the surface sinks because of its greater density and is replaced by warmer water from below. This mixing continues until the temperature reaches 4°C. As the surface water cools further, it remains on the surface because it is less dense than the 4°C water below. Water then freezes first at the surface, and the ice remains on the surface since ice (specific gravity = 0.917) is less dense than water. The water at the bottom remains liquid unless it is so cold that the whole body of water freezes. If water were like most substances, becoming more dense as it cools, the water at the bottom of a lake would be frozen first. Lakes would freeze solid more easily since circulation would bring the warmer water to the surface to be efficiently cooled. The complete freezing of a lake would cause severe damage to its plant and animal life. Because of the unusual behavior of water below 4°C, it is rare for any large body of water to freeze completely, and this is helped by the layer of ice on the surface which acts as an insulator to reduce the flow of heat out of the water into the cold air above. Without this peculiar but wonderful property of water, life on this planet as we know it might not have been possible.

PHYSICS APPLIED
Life under ice

Not only does water expand as it cools from 4°C to 0°C, it expands even more as it freezes to ice. This is why ice cubes float in water and pipes break when water inside them freezes.

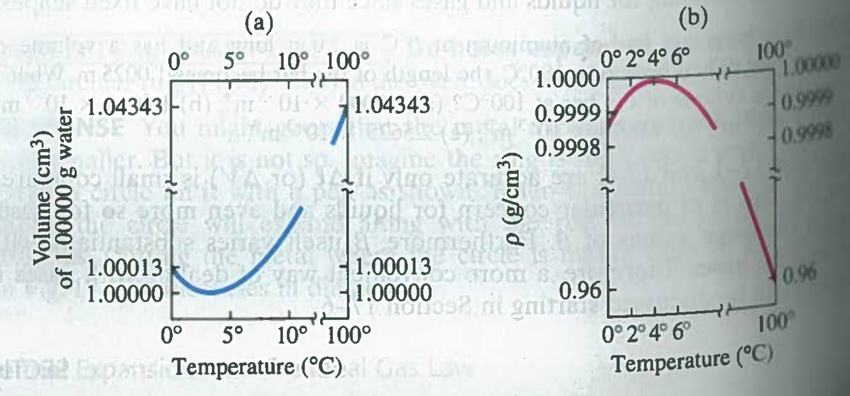


FIGURE 17–12 Behavior of water as a function of temperature near 4°C. (a) Volume of 1.00000 g of water, as a function of temperature. (b) Density vs. temperature. [Note the break in each axis.]

17–5 Thermal Stresses

In many situations, such as in buildings and roads, the ends of a beam or slab of material are rigidly fixed, which greatly limits expansion or contraction. If the temperature should change, large compressive or tensile stresses, called *thermal stresses*, will occur. The magnitude of such stresses can be calculated using the concept of elastic modulus developed in Chapter 12. To calculate the internal stress, we can think of this process as occurring in two steps: (1) the beam tries to expand (or contract) by an amount $\Delta \ell$ given by Eq. 17–1; (2) the solid in contact with the beam exerts a force to compress (or expand) it, keeping it at its original length. The force F required is given by Eq. 12–4:

$$\Delta \ell = \frac{1}{E}\frac{F}{A}\ell_0,$$

where E is Young's modulus for the material. To calculate the internal stress, F/A, we then set $\Delta \ell$ in Eq. 17–1a equal to $\Delta \ell$ in the equation above and find

$$\alpha \ell_0 \Delta T = \frac{1}{E}\frac{F}{A}\ell_0.$$

Hence, the stress

$$\frac{F}{A} = \alpha E \, \Delta T.$$

EXAMPLE 17–8 **Stress in concrete on a hot day.** A highway is to be made of blocks of concrete 10 m long placed end to end with no space between them to allow for expansion. If the blocks were placed at a temperature of 10°C, what compressive stress would occur if the temperature reached 40°C? The contact area between each block is 0.20 m². Will fracture occur?

PHYSICS APPLIED
Highway buckling

APPROACH We use the expression for the stress F/A we just derived, and find the value of E from Table 12–1. To see if fracture occurs, we compare this stress to the ultimate strength of concrete in Table 12–2.

SOLUTION

$$\frac{F}{A} = \alpha E \, \Delta T = (12 \times 10^{-6}/\mathrm{C}^\circ)(20 \times 10^{9}\,\mathrm{N/m^2})(30\,\mathrm{C}^\circ) = 7.2 \times 10^{6}\,\mathrm{N/m^2}.$$

This stress is not far from the ultimate strength of concrete under compression (Table 12–2) and exceeds it for tension and shear. If the concrete is not perfectly aligned, part of the force will act in shear, and fracture is likely. This is why soft spacers or expansion joints (Fig. 17–3) are used in concrete sidewalks, highways, and bridges.

EXERCISE C How much space would you allow between the 10-m-long concrete blocks if you expected a temperature range of 0°F to 110°F?

17–6 The Gas Laws and Absolute Temperature

Equation 17–2 is not very useful for describing the expansion of a gas, partly because the expansion can be so great, and partly because gases generally expand to fill whatever container they are in. Indeed, Eq. 17–2 is meaningful only if the pressure is kept constant. The volume of a gas depends very much on the pressure as well as on the temperature. It is therefore valuable to determine a relation between the volume, the pressure, the temperature, and the mass of a gas. Such a relation is called an **equation of state**. (By the word *state*, we mean the physical condition of the system.)

If the state of a system is changed, we will always wait until the pressure and temperature have reached the same values throughout. We thus consider only **equilibrium states** of a system—when the variables that describe it (such as temperature and pressure) are the same throughout the system and are not changing in time. We also note that the results of this Section are accurate only for gases that are not too dense (the pressure is not too high, on the order of an atmosphere or less) and not close to the liquefaction (boiling) point.

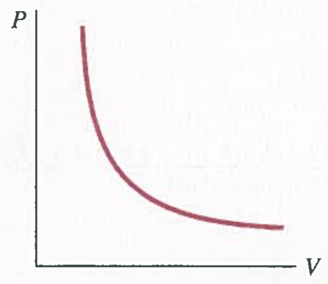
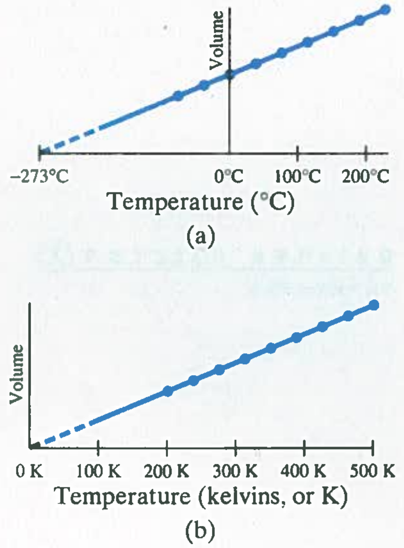


FIGURE 17–13 Pressure vs. volume of a fixed amount of gas at a constant temperature, showing the inverse relationship as given by Boyle's law: as the pressure decreases, the volume increases.

FIGURE 17–14 Volume of a fixed amount of gas as a function of (a) Celsius temperature, and (b) Kelvin temperature, when the pressure is kept constant.



For a given quantity of gas it is found experimentally that, to a good approximation, *the volume of a gas is inversely proportional to the absolute pressure applied to it when the temperature is kept constant*. That is,

$$V \propto \frac{1}{P}, \qquad [\text{constant } T]$$

where P is the absolute pressure (*not* "gauge pressure"—see Section 13–4). For example, if the pressure on a gas is doubled, the volume is reduced to half its original volume. This relation is known as **Boyle's law**, after Robert Boyle (1627–1691), who first stated it on the basis of his own experiments. A graph of P vs. V for a fixed temperature is shown in Fig. 17–13. Boyle's law can also be written

$$PV = \text{constant}. \qquad [\text{constant } T]$$

That is, at constant temperature, if either the pressure or volume of a fixed amount of gas is allowed to vary, the other variable also changes so that the product PV remains constant.

Temperature also affects the volume of a gas, but a quantitative relationship between V and T was not found until more than a century after Boyle's work. The Frenchman Jacques Charles (1746–1823) found that when the pressure is not too high and is kept constant, the volume of a gas increases with temperature at a nearly linear rate, as shown in Fig. 17–14a. However, all gases liquefy at low temperatures (for example, oxygen liquefies at −183°C), so the graph cannot be extended below the liquefaction point. Nonetheless, the graph is essentially a straight line and if projected to lower temperatures, as shown by the dashed line, it crosses the axis at about −273°C.

Such a graph can be drawn for any gas, and the straight line always projects back to −273°C at zero volume. This seems to imply that if a gas could be cooled to −273°C, it would have zero volume, and at lower temperatures a negative volume, which makes no sense. It could be argued that −273°C is the lowest temperature possible; indeed, many other more recent experiments indicate that this is so. This temperature is called the **absolute zero** of temperature. Its value has been determined to be −273.15°C.

Absolute zero forms the basis of a temperature scale known as the **absolute scale** or **Kelvin scale**, and it is used extensively in scientific work. On this scale the temperature is specified as degrees Kelvin or, preferably, simply as *kelvins* (K) without the degree sign. The intervals are the same as for the Celsius scale, but the zero on this scale (0 K) is chosen as absolute zero. Thus the freezing point of water (0°C) is 273.15 K, and the boiling point of water is 373.15 K. Indeed, any temperature on the Celsius scale can be changed to kelvins by adding 273.15 to it:

$$T(\text{K}) = T(^\circ\text{C}) + 273.15.$$

Now let us look at Fig. 17–14b, where the graph of the volume of a gas versus absolute temperature is a straight line that passes through the origin. Thus, to a good approximation, *the volume of a given amount of gas is directly proportional to the absolute temperature when the pressure is kept constant*. This is known as **Charles's law**, and is written

$$V \propto T. \qquad [\text{constant } P]$$

A third gas law, known as **Gay-Lussac's law**, after Joseph Gay-Lussac (1778–1850), states that *at constant volume, the absolute pressure of a gas is directly proportional to the absolute temperature*:

$$P \propto T. \qquad [\text{constant } V]$$

The laws of Boyle, Charles, and Gay-Lussac are not really laws in the sense that we use this term today (precise, deep, wide-ranging validity). They are really only approximations that are accurate for real gases only as long as the pressure and density of the gas are not too high, and the gas is not too close to liquefaction (condensation). The term *law* applied to these three relationships has become traditional, however, so we have stuck with that usage.

CONCEPTUAL EXAMPLE 17–9 **Why you should not throw a closed glass jar into a campfire.** What can happen if you did throw an empty glass jar, with the lid on tight, into a fire, and why?

RESPONSE The inside of the jar is not empty. It is filled with air. As the fire heats the air inside, its temperature rises. The volume of the glass jar changes only slightly due to the heating. According to Gay-Lussac's law the pressure P of the air inside the jar can increase dramatically, enough to cause the jar to explode, throwing glass pieces outward.

17–7 The Ideal Gas Law

The gas laws of Boyle, Charles, and Gay-Lussac were obtained by means of a very useful scientific technique: namely, holding one or more variables constant to see clearly the effects on one variable due to changing one other variable. These laws can now be combined into a single more general relation between the absolute pressure, volume, and absolute temperature of a fixed quantity of gas:

$$PV \propto T.$$

This relation indicates how any of the quantities P, V, or T will vary when the other two quantities change. This relation reduces to Boyle's, Charles's, or Gay-Lussac's law when either T, P, or V, respectively, is held constant.

Finally, we must incorporate the effect of the amount of gas present. Anyone who has blown up a balloon knows that the more air forced into the balloon, the bigger it gets (Fig. 17–15). Indeed, careful experiments show that at constant temperature and pressure, the volume V of an enclosed gas increases in direct proportion to the mass m of gas present. Hence we write

$$PV \propto mT.$$

FIGURE 17–15 Blowing up a balloon means putting more air (more air molecules) into the balloon, which increases its volume. The pressure is nearly constant (atmospheric) except for the small effect of the balloon's elasticity.

This proportion can be made into an equation by inserting a constant of proportionality. Experiment shows that this constant has a different value for different gases. However, the constant of proportionality turns out to be the same for all gases if, instead of the mass m, we use the number of *moles*.

One **mole** (abbreviated mol) is defined as the amount of substance that contains as many atoms or molecules as there are in precisely 12 grams of carbon 12 (whose atomic mass is exactly 12 u). A simpler but equivalent definition is this: 1 mol is that quantity of substance whose mass in grams is numerically equal to the molecular mass (Section 17–1) of the substance. For example, the molecular mass of hydrogen gas (H_2) is 2.0 u (since each molecule contains two atoms of hydrogen and each atom has an atomic mass of 1.0 u). Thus 1 mol of H_2 has a mass of 2.0 g. Similarly, 1 mol of neon gas has a mass of 20 g, and 1 mol of CO_2 has a mass of $[12 + (2 \times 16)] = 44$ g since oxygen has atomic mass of 16 (see Periodic Table inside the rear cover). The mole is the official unit of amount of substance in the SI system. In general, the number of moles, n, in a given sample of a pure substance is equal to the mass of the sample in grams divided by the molecular mass specified as grams per mole:

$$n\,(\text{mole}) = \frac{\text{mass (grams)}}{\text{molecular mass (g/mol)}}.$$

For example, the number of moles in 132 g of CO_2 (molecular mass 44 u) is

$$n = \frac{132\text{ g}}{44\text{ g/mol}} = 3.0\text{ mole}.$$

We can now write the proportion discussed above ($PV \propto mT$) as an equation:

IDEAL GAS LAW

$$PV = nRT, \qquad (17\text{–}3)$$

where n represents the number of moles and R is the constant of proportionality. R is called the **universal gas constant** because its value is found experimentally to be the same for all gases. The value of R, in several sets of units (only the first is the proper SI unit), is

$$\begin{aligned} R &= 8.314\ \text{J/(mol·K)} && \text{[SI units]} \\ &= 0.0821\ \text{(L·atm)/(mol·K)} \\ &= 1.99\ \text{calories/(mol·K)}.^{\dagger} \end{aligned}$$

Equation 17–3 is called the **ideal gas law**, or the **equation of state for an ideal gas**. We use the term "ideal" because real gases do not follow Eq. 17–3 precisely, particularly at high pressure (and density) or when the gas is near the liquefaction point (= boiling point). However, at pressures less than an atmosphere or so, and when T is not close to the liquefaction point of the gas, Eq. 17–3 is quite accurate and useful for real gases.

CAUTION

Always give T in kelvins and P as absolute (not gauge) pressure

Always remember, when using the ideal gas law, that temperatures must be given in kelvins (K) and that the pressure P must always be *absolute* pressure, not gauge pressure (Section 13–4).

EXERCISE D Return to the Chapter-Opening Question, page 454, and answer it again now. Try to explain why you may have answered differently the first time.

EXERCISE E An ideal gas is contained in a steel sphere at 27.0°C and 1.00 atm absolute pressure. If no gas is allowed to escape and the temperature is raised to 127°C, what will be the new pressure? (*a*) 1.33 atm; (*b*) 0.75 atm; (*c*) 4.7 atm; (*d*) 0.21 atm; (*e*) 1.00 atm.

17–8 Problem Solving with the Ideal Gas Law

The ideal gas law is an extremely useful tool, and we now consider some Examples. We will often refer to "standard conditions" or **standard temperature and pressure (STP)**, which means:

STP

$$T = 273\ \text{K}\ (0°\text{C}) \quad \text{and} \quad P = 1.00\ \text{atm} = 1.013 \times 10^5\ \text{N/m}^2 = 101.3\ \text{kPa}.$$

EXAMPLE 17–10 Volume of one mole at STP. Determine the volume of 1.00 mol of any gas, assuming it behaves like an ideal gas, at STP.

APPROACH We use the ideal gas law, solving for V.

SOLUTION We solve for V in Eq. 17–3:

$$V = \frac{nRT}{P} = \frac{(1.00\ \text{mol})(8.314\ \text{J/mol·K})(273\ \text{K})}{(1.013 \times 10^5\ \text{N/m}^2)} = 22.4 \times 10^{-3}\ \text{m}^3.$$

Since 1 liter (L) is $1000\ \text{cm}^3 = 1.00 \times 10^{-3}\ \text{m}^3$, 1.00 mol of any (ideal) gas has volume $V = 22.4\ \text{L}$ at STP.

PROBLEM SOLVING

1 mol of gas at STP has V = 22.4 L

The value of 22.4 L for the volume of 1 mol of an ideal gas at STP is worth remembering, for it sometimes makes calculation simpler.

EXERCISE F What is the volume of 1.00 mol of ideal gas at 546 K (= 2 × 273 K) and 2.0 atm absolute pressure? (*a*) 11.2 L, (*b*) 22.4 L, (*c*) 44.8 L, (*d*) 67.2 L, (*e*) 89.6 L.

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†Calories will be defined in Section 19–1; sometimes it is useful to use R as given in terms of calories.

EXAMPLE 17–11 Helium balloon. A helium party balloon, assumed to be a perfect sphere, has a radius of 18.0 cm. At room temperature (20°C), its internal pressure is 1.05 atm. Find the number of moles of helium in the balloon and the mass of helium needed to inflate the balloon to these values.

APPROACH We can use the ideal gas law to find n, since we are given P and T, and can find V from the given radius.

SOLUTION We get the volume V from the formula for a sphere:

$$\begin{aligned} V &= \tfrac{4}{3}\pi r^3 \\ &= \tfrac{4}{3}\pi\,(0.180\ \text{m})^3 = 0.0244\ \text{m}^3. \end{aligned}$$

The pressure is given as $1.05\ \text{atm} = 1.064 \times 10^5\ \text{N/m}^2$. The temperature must be expressed in kelvins, so we change 20°C to $(20 + 273)\text{K} = 293\ \text{K}$. Finally, we use the value $R = 8.314\ \text{J/(mol·K)}$ because we are using SI units. Thus

$$n = \frac{PV}{RT} = \frac{(1.064 \times 10^5\ \text{N/m}^2)(0.0244\ \text{m}^3)}{(8.314\ \text{J/mol·K})(293\ \text{K})} = 1.066\ \text{mol}.$$

The mass of helium (atomic mass = 4.00 g/mol as given in the Periodic Table or Appendix F) can be obtained from

$$\text{mass} = n \times \text{molecular mass} = (1.066\ \text{mol})(4.00\ \text{g/mol}) = 4.26\ \text{g}$$

or 4.26×10^{-3} kg.

EXAMPLE 17–12 ESTIMATE Mass of air in a room. Estimate the mass of air in a room whose dimensions are 5 m × 3 m × 2.5 m high, at STP.

APPROACH First we determine the number of moles n using the given volume. Then we can multiply by the mass of one mole to get the total mass.

SOLUTION Example 17–10 told us that 1 mol of a gas at 0°C has a volume of 22.4 L. The room's volume is 5 m × 3 m × 2.5 m, so

$$n = \frac{(5\ \text{m})(3\ \text{m})(2.5\ \text{m})}{22.4 \times 10^{-3}\ \text{m}^3} \approx 1700\ \text{mol}.$$

Air is a mixture of about 20% oxygen (O_2) and 80% nitrogen (N_2). The molecular masses are $2 \times 16\ \text{u} = 32\ \text{u}$ and $2 \times 14\ \text{u} = 28\ \text{u}$, respectively, for an average of about 29 u. Thus, 1 mol of air has a mass of about $29\ \text{g} = 0.029\ \text{kg}$, so our room has a mass of air

$$m \approx (1700\ \text{mol})(0.029\ \text{kg/mol}) \approx 50\ \text{kg}.$$

NOTE That is roughly 100 lb of air!

PHYSICS APPLIED

Mass (and weight) of the air in a room

EXERCISE G At 20°C, would there be (*a*) more, (*b*) less, or (*c*) the same air mass in a room than at 0°C?

Frequently, volume is specified in liters and pressure in atmospheres. Rather than convert these to SI units, we can instead use the value of R given in Section 17–7 as 0.0821 L·atm/mol·K.

PROBLEM SOLVING

Using the ideal gas law as a ratio

In many situations it is not necessary to use the value of R at all. For example, many problems involve a change in the pressure, temperature, and volume of a fixed amount of gas. In this case, $PV/T = nR = \text{constant}$, since n and R remain constant. If we now let P_1, V_1, and T_1 represent the appropriate variables initially, and P_2, V_2, T_2 represent the variables after the change is made, then we can write

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}.$$

If we know any five of the quantities in this equation, we can solve for the sixth. Or, if one of the three variables is constant ($V_1 = V_2$, or $P_1 = P_2$, or $T_1 = T_2$) then we can use this equation to solve for one unknown when given the other three quantities.

PHYSICS APPLIED
Pressure in a hot tire

FIGURE 17–16 Example 17–13.

EXAMPLE 17–13 **Check tires cold.** An automobile tire is filled (Fig. 17–16) to a gauge pressure of 200 kPa at 10°C. After a drive of 100 km, the temperature within the tire rises to 40°C. What is the pressure within the tire now?

APPROACH We do not know the number of moles of gas, or the volume of the tire, but we assume they are constant. We use the ratio form of the ideal gas law.

SOLUTION Since $V_1 = V_2$, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}.$$

This is, incidentally, a statement of Gay-Lussac's law. Since the pressure given is the gauge pressure (Section 13–4), we must add atmospheric pressure (= 101 kPa) to get the absolute pressure $P_1 = (200\,\text{kPa} + 101\,\text{kPa}) = 301\,\text{kPa}$. We convert temperatures to kelvins by adding 273 and solve for P_2:

$$P_2 = P_1\left(\frac{T_2}{T_1}\right) = (3.01 \times 10^5\,\text{Pa})\left(\frac{313\,\text{K}}{283\,\text{K}}\right) = 333\,\text{kPa}.$$

Subtracting atmospheric pressure, we find the resulting gauge pressure to be 232 kPa, which is a 16% increase. This Example shows why car manuals suggest checking tire pressure when the tires are cold.

17–9 Ideal Gas Law in Terms of Molecules: Avogadro's Number

The fact that the gas constant, R, has the same value for all gases is a remarkable reflection of simplicity in nature. It was first recognized, although in a slightly different form, by the Italian scientist Amedeo Avogadro (1776–1856). Avogadro stated that *equal volumes of gas at the same pressure and temperature contain equal numbers of molecules*. This is sometimes called **Avogadro's hypothesis**. That this is consistent with R being the same for all gases can be seen as follows. From Eq. 17–3, $PV = nRT$, we see that for the same number of moles, n, and the same pressure and temperature, the volume will be the same for all gases as long as R is the same. Second, the number of molecules in 1 mole is the same for all gases. Thus Avogadro's hypothesis is equivalent to R being the same for all gases.

Avogadro's number

The number of molecules in one mole of any pure substance is known as **Avogadro's number**, N_A. Although Avogadro conceived the notion, he was not able to actually determine the value of N_A. Indeed, precise measurements were not done until the twentieth century.

A number of methods have been devised to measure N_A, and the accepted value today is

$$N_A = 6.02 \times 10^{23}. \qquad \text{[molecules/mole]}$$

Since the total number of molecules, N, in a gas is equal to the number per mole times the number of moles ($N = nN_A$), the ideal gas law, Eq. 17–3, can be written in terms of the number of molecules present:

$$PV = nRT = \frac{N}{N_A}RT,$$

or

IDEAL GAS LAW (in terms of molecules)

$$PV = NkT, \qquad (17\text{–}4)$$

where $k = R/N_A$ is called the **Boltzmann constant** and has the value

$$k = \frac{R}{N_A} = \frac{8.314\,\text{J/mol}\cdot\text{K}}{6.02 \times 10^{23}/\text{mol}} = 1.38 \times 10^{-23}\,\text{J/K}.$$

EXAMPLE 17–14 **Hydrogen atom mass.** Use Avogadro's number to determine the mass of a hydrogen atom.

APPROACH The mass of one atom equals the mass of 1 mol divided by the number of atoms in 1 mol, N_A.

SOLUTION One mole of hydrogen atoms (atomic mass = 1.008 u, Section 17–1 or Appendix F) has a mass of 1.008×10^{-3} kg and contains 6.02×10^{23} atoms. Thus one atom has a mass

$$m = \frac{1.008 \times 10^{-3}\,\text{kg}}{6.02 \times 10^{23}}$$

$$= 1.67 \times 10^{-27}\,\text{kg}.$$

PHYSICS APPLIED
Molecules in a breath

EXAMPLE 17–15 **ESTIMATE** **How many molecules in one breath?** Estimate how many molecules you breathe in with a 1.0-L breath of air.

APPROACH We determine what fraction of a mole 1.0 L is by using the result of Example 17–10 that 1 mole has a volume of 22.4 L at STP, and then multiply that by N_A to get the number of molecules in this number of moles.

SOLUTION One mole corresponds to 22.4 L at STP, so 1.0 L of air is (1.0 L)/(22.4 L/mol) = 0.045 mol. Then 1.0 L of air contains

$$(0.045\,\text{mol})(6.02 \times 10^{23}\,\text{molecules/mol}) \approx 3 \times 10^{22}\,\text{molecules}.$$

*17–10 Ideal Gas Temperature Scale—a Standard

It is important to have a very precisely defined temperature scale so that measurements of temperature made at different laboratories around the world can be accurately compared. We now discuss such a scale that has been accepted by the general scientific community.

The standard thermometer for this scale is the constant-volume gas thermometer discussed in Section 17–2. The scale itself is called the **ideal gas temperature scale**, since it is based on the property of an ideal gas that the pressure is directly proportional to the absolute temperature (Gay-Lussac's law). A real gas, which would need to be used in any real constant-volume gas thermometer, approaches this ideal at low density. In other words, the temperature at any point in space is *defined* as being proportional to the pressure in the (nearly) ideal gas used in the thermometer. To set up a scale we need two fixed points. One fixed point will be $P = 0$ at $T = 0$ K. The second fixed point is chosen to be the **triple point** of water, which is that point where water in the solid, liquid, and gas states can coexist in equilibrium. This occurs only at a unique temperature and pressure,[†] and can be reproduced at different laboratories with great precision. The pressure at the triple point of water is 4.58 torr and the temperature is 0.01°C. This temperature corresponds to 273.16 K, since absolute zero is about −273.15°C. In fact, the triple point is now *defined* to be exactly 273.16 K.

[†] For example, the molecular mass of H_2 gas is 2.0 atomic mass units (u), whereas that of O_2 gas is 32.0 u. Thus 1 mol of H_2 has a mass of 0.0020 kg and 1 mol of O_2 gas, 0.0320 kg. The number of molecules in a mole is equal to the total mass M of a mole divided by the mass m of one molecule; since the ratio (M/m) is the same for all gases by definition of the mole, a mole of any gas must contain the same number of molecules.

[†] Liquid water and steam can coexist (the boiling point) at a range of temperatures depending on the pressure. Water boils at a lower temperature when the pressure is less, such as high in the mountains. The triple point represents a more precisely reproducible fixed point than does either the freezing point or boiling point of water at, say, 1 atm. See Section 18–3 for further discussion.

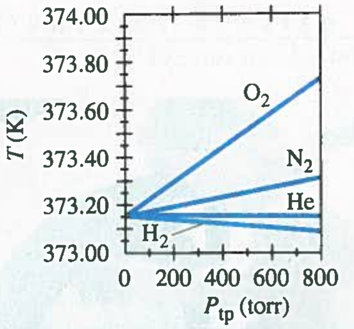

FIGURE 17–17 Temperature readings of a constant-volume gas thermometer for the boiling point of water at 1.00 atm are plotted, for different gases, as a function of the gas pressure in the thermometer at the triple point (P_{tp}). Note that as the amount of gas in the thermometer is reduced, so that $P_{tp} \to 0$, all gases give the same reading, 373.15 K. For pressure less than 0.10 atm (76 torr), the variation shown is less than 0.07 K.

The absolute or Kelvin temperature T at any point is then defined, using a constant-volume gas thermometer for an ideal gas, as

$$T = (273.16\,\text{K})\left(\frac{P}{P_{tp}}\right). \qquad \text{[ideal gas; constant volume]} \quad (17\text{–}5a)$$

In this relation, P_{tp} is the pressure of the gas in the thermometer at the triple point temperature of water, and P is the pressure in the thermometer when it is at the point where T is being determined. Note that if we let $P = P_{tp}$ in this relation, then $T = 273.16\,\text{K}$, as it must.

The definition of temperature, Eq. 17–5a, with a constant-volume gas thermometer filled with a real gas is only approximate because we find that we get different results for the temperature depending on the type of gas that is used in the thermometer. Temperatures determined in this way also vary depending on the amount of gas in the bulb of the thermometer: for example, the boiling point of water at 1.00 atm is found from Eq. 17–5a to be 373.87 K when the gas is O_2 and $P_{tp} = 1000$ torr. If the amount of O_2 in the bulb is reduced so that at the triple point $P_{tp} = 500$ torr, the boiling point of water from Eq. 17–5a is then found to be 373.51 K. If H_2 gas is used instead, the corresponding values are 373.07 K and 373.11 K (see Fig. 17–17). But now suppose we use a particular real gas and make a series of measurements in which the amount of gas in the thermometer bulb is reduced to smaller and smaller amounts, so that P_{tp} becomes smaller and smaller. It is found experimentally that an extrapolation of such data to $P_{tp} = 0$ always gives the *same value* for the temperature of a given system (such as $T = 373.15\,\text{K}$ for the boiling point of water at 1.00 atm) as shown in Fig. 17–17. Thus the temperature T at any point in space, determined using a constant-volume gas thermometer containing a real gas, is defined using this limiting process:

$$T = (273.16\,\text{K}) \lim_{P_{tp}\to 0}\left(\frac{P}{P_{tp}}\right). \qquad \text{[constant volume]} \quad (17\text{–}5b)$$

This defines the **ideal gas temperature scale**. One of the great advantages of this scale is that the value for T does not depend on the kind of gas used. But the scale does depend on the properties of gases in general. Helium has the lowest condensation point of all gases; at very low pressures it liquefies at about 1 K, so temperatures below this cannot be defined on this scale.

Summary

The atomic theory of matter postulates that all matter is made up of tiny entities called **atoms**, which are typically 10^{-10} m in diameter.

Atomic and **molecular masses** are specified on a scale where ordinary carbon (^{12}C) is arbitrarily given the value 12.0000 u (atomic mass units).

The distinction between solids, liquids, and gases can be attributed to the strength of the attractive forces between the atoms or molecules and to their average speed.

Temperature is a measure of how hot or cold something is. **Thermometers** are used to measure temperature on the **Celsius** (°C), **Fahrenheit** (°F), and **Kelvin** (K) scales. Two standard points on each scale are the freezing point of water (0°C, 32°F, 273.15 K) and the boiling point of water (100°C, 212°F, 373.15 K). A one-kelvin change in temperature equals a change of one Celsius degree or $\frac{9}{5}$ Fahrenheit degrees. Kelvins are related to °C by $T(\text{K}) = T(°\text{C}) + 273.15$.

The change in length, $\Delta\ell$, of a solid, when its temperature changes by an amount ΔT, is directly proportional to the temperature change and to its original length ℓ_0. That is,

$$\Delta\ell = \alpha\ell_0\,\Delta T, \qquad (17\text{–}1a)$$

where α is the *coefficient of linear expansion*.

The change in volume of most solids, liquids, and gases is proportional to the temperature change and to the original volume V_0:

$$\Delta V = \beta V_0\,\Delta T. \qquad (17\text{–}2)$$

The *coefficient of volume expansion*, β, is approximately equal to 3α for uniform solids.

Water is unusual because, unlike most materials whose volume increases with temperature, its volume actually decreases as the temperature increases in the range from 0°C to 4°C.

The **ideal gas law**, or **equation of state for an ideal gas**, relates the pressure P, volume V, and temperature T (in kelvins) of n moles of gas by the equation

$$PV = nRT, \qquad (17\text{–}3)$$

where $R = 8.314\,\text{J/mol}\cdot\text{K}$ for all gases. Real gases obey the ideal gas law quite accurately if they are not at too high a pressure or near their liquefaction point.

One **mole** is that amount of a substance whose mass in grams is numerically equal to the atomic or molecular mass of that substance.

Avogadro's number, $N_A = 6.02 \times 10^{23}$, is the number of atoms or molecules in 1 mol of any pure substance.

The ideal gas law can be written in terms of the number of molecules N in the gas as

$$PV = NkT, \qquad (17\text{–}4)$$

where $k = R/N_A = 1.38 \times 10^{-23}\,\text{J/K}$ is Boltzmann's constant.

Questions

1. Which has more atoms: 1 kg of iron or 1 kg of aluminum? See the Periodic Table or Appendix F.
2. Name several properties of materials that could be exploited to make a thermometer.
3. Which is larger, 1C° or 1F°?
4. If system A is in equilibrium with system B, but B is not in equilibrium with system C, what can you say about the temperatures of A, B, and C?
5. Suppose system C is not in equilibrium with system A nor in equilibrium with system B. Does this imply that A and B are not in equilibrium? What can you infer regarding the temperatures of A, B, and C?
6. In the relation $\Delta\ell = \alpha\ell_0\,\Delta T$, should ℓ_0 be the initial length, the final length, or does it matter?
7. A flat bimetallic strip consists of a strip of aluminum riveted to a strip of iron. When heated, the strip will bend. Which metal will be on the outside of the curve? Why?
8. Long steam pipes that are fixed at the ends often have a section in the shape of a U. Why?
9. A flat uniform cylinder of lead floats in mercury at 0°C. Will the lead float higher or lower if the temperature is raised?
10. Figure 17–18 shows a diagram of a simple **thermostat** used to control a furnace (or other heating or cooling system). The bimetallic strip consists of two strips of different metals bonded together. The electric switch (attached to the bimetallic strip) is a glass vessel containing liquid mercury that conducts electricity when it can flow to touch both contact wires. Explain how this device controls the furnace and how it can be set at different temperatures.

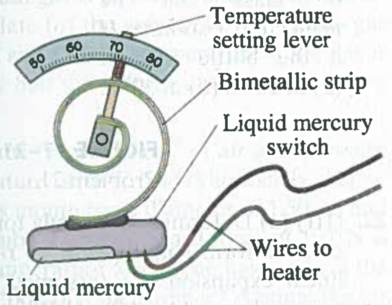

FIGURE 17–18 A thermostat (Question 10).

11. Explain why it is advisable to add water to an overheated automobile engine only slowly, and only with the engine running.
12. The units for the coefficients of expansion α are $(\text{C}°)^{-1}$, and there is no mention of a length unit such as meters. Would the expansion coefficient change if we used feet or millimeters instead of meters?
13. When a cold mercury-in-glass thermometer is first placed in a hot tub of water, the mercury initially descends a bit and then rises. Explain.
14. The principal virtue of Pyrex glass is that its coefficient of linear expansion is much smaller than that for ordinary glass (Table 17–1). Explain why this gives rise to the higher resistance to heat of Pyrex.
15. Will a grandfather clock, accurate at 20°C, run fast or slow on a hot day (30°C)? The clock uses a pendulum supported on a long thin brass rod.
16. Freezing a can of soda will cause its bottom and top to bulge so badly the can will not stand up. What has happened?
17. Why might you expect an alcohol-in-glass thermometer to be more precise than a mercury-in-glass thermometer?
18. Will the buoyant force on an aluminum sphere submerged in water increase, decrease, or remain the same, if the temperature is increased from 20°C to 40°C?
19. If an atom is measured to have a mass of 6.7×10^{-27} kg, what atom do you think it is?
20. From a practical point of view, does it really matter what gas is used in a constant-volume gas thermometer? If so, explain. [*Hint*: See Fig. 17–17.]
21. A ship loaded in sea water at 4°C later sailed up a river into fresh water where it sank in a storm. Explain why a ship might be more likely to sink in fresh water than on the open sea. [*Hint*: Consider the buoyant force due to water.]

Problems

17–1 Atomic Theory

1. (I) How does the number of atoms in a 21.5-g gold ring compare to the number in a silver ring of the same mass?
2. (I) How many atoms are there in a 3.4-g copper penny?

17–2 Temperature and Thermometers

3. (I) (*a*) "Room temperature" is often taken to be 68°F. What is this on the Celsius scale? (*b*) The temperature of the filament in a lightbulb is about 1900°C. What is this on the Fahrenheit scale?
4. (I) Among the highest and lowest natural air temperatures recorded are 136°F in the Libyan desert and −129°F in Antarctica. What are these temperatures on the Celsius scale?
5. (I) A thermometer tells you that you have a fever of 39.4°C. What is this in Fahrenheit?
6. (II) In an alcohol-in-glass thermometer, the alcohol column has length 11.82 cm at 0.0°C and length 21.85 cm at 100.0°C. What is the temperature if the column has length (*a*) 18.70 cm, and (*b*) 14.60 cm?

17–4 Thermal Expansion

7. (I) The Eiffel Tower (Fig. 17–19) is built of wrought iron approximately 300 m tall. Estimate how much its height changes between January (average temperature of 2°C) and July (average temperature of 25°C). Ignore the angles of the iron beams and treat the tower as a vertical beam.

FIGURE 17–19 Problem 7. The Eiffel Tower in Paris.