

More applications of D&C

9/19/24

Administrivia

- Read Section 9.3 for tomorrow
- Expect HW out tonight

Recall: Multiplying large integers

$$\begin{array}{r} 782934728937492347982378942 \\ \times \quad 3789234783974983274832792 \\ \hline \end{array}$$

Represent numbers as arrays of digits

Grade-school multiplication is $O(n^2)$

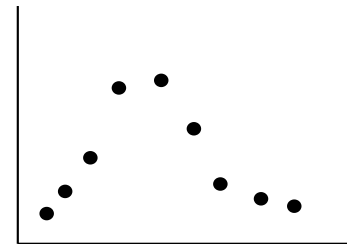
Karatsuba algorithm (1962)

Practice problem 1

(K&T solved problem 5.1)

Suppose you have an array A of n distinct numbers and that the sequence A_1, A_2, \dots, A_n is *unimodal*, meaning it increases up to some index and then decreases.

| | | | | | | | |
|---|---|----|----|----|----|---|----|
| 3 | 5 | 23 | 27 | 26 | 18 | 5 | -1 |
|---|---|----|----|----|----|---|----|



Develop an algorithm to find the index of the largest value while examining only $O(\log n)$ entries of A .

Practice problem 2

Given an array of n sorted integers, find the indices of the first and last occurrences of a value x in $O(\log n)$ time.

Practice problem 3

Suppose the time to merge two sorted lists is equal to their combined length. How long does it take to merge lists L_1 , L_2 , L_3 , and L_4 containing m values each with the code below?

```
for(i=2; i<=4; i++)
```

```
     $L_1 = \text{merge}(L_1, L_i);$ 
```

A. $4m$

B. $6m$

C. $8m$

D. $9m$

E. None of the above

Practice problem 3

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Practice problem 3

What is the time to combine k lists, each containing m values, using this same algorithm?

What is a better algorithm and its running time?