Matrix chain multiplication

(and dynamic programming practice)

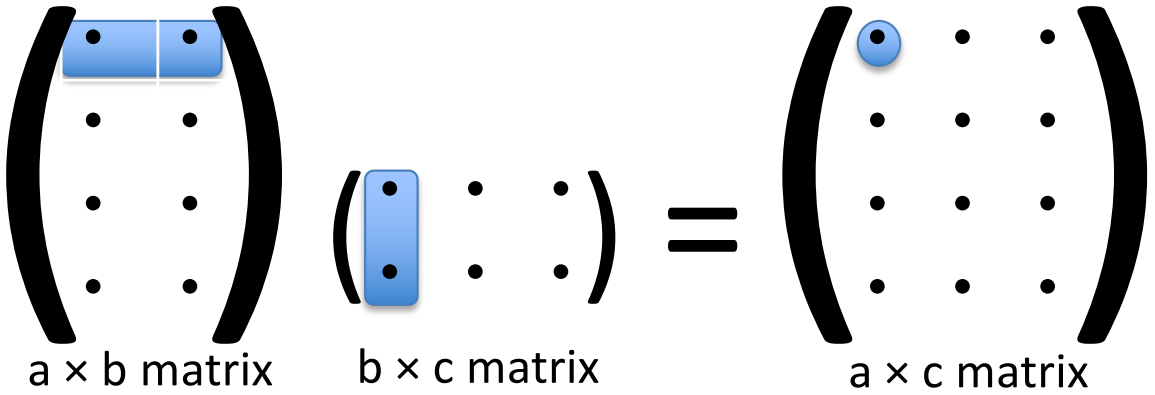
10/4/24

(keep working on the exam)

Due Tuesday night

Matrix chain multiplication

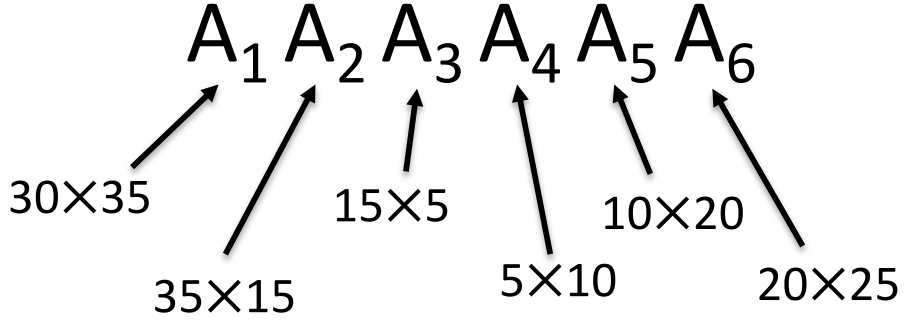
- Can multiply non-square matrices:



- Number of operations = abc (using simple alg)
- How long does it take to multiply a series ("chain") of them?

Problem instances

How should we multiply the following?



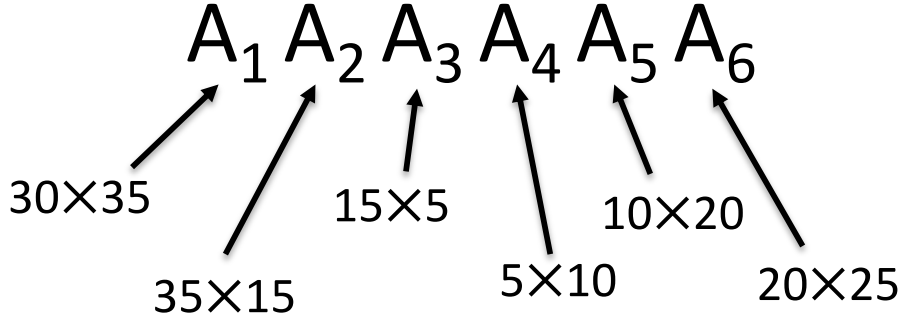
Representation of problem:

$p[0..6] = (30, 35, 15, 5, 10, 20, 25)$

Matrix A_i has dimensions $p_{i-1} \times p_i$

Problem instances

How should we multiply the following?



Show that neither of the following work in general:

- Perform the single cheapest multiplication first
- Perform the multiplication giving the smallest-dimensioned result first (smallest dimension product)

Why not try them all?

P_n = # of ways to group n items with parentheses

$P_1 = P_2 = 1, P_3 = 2$

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What is P_4?

A. 2

B. 3

C. 4

D. 5

E. None of the above

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General formula:

$$P_n = \sum_{k=1}^{n-1} P_k P_{n-k}$$

$P_4 = 5$

$P_5 = 14$

$P_6 = 42$

$P_7 = 132$

$P_8 = 429$

Finding a subproblem

- What is the optimal number of multiplications to combine a range of matrices?

(30, 35, 15, 5, 10, 20, 25)

0 1 2 3 4 5 6

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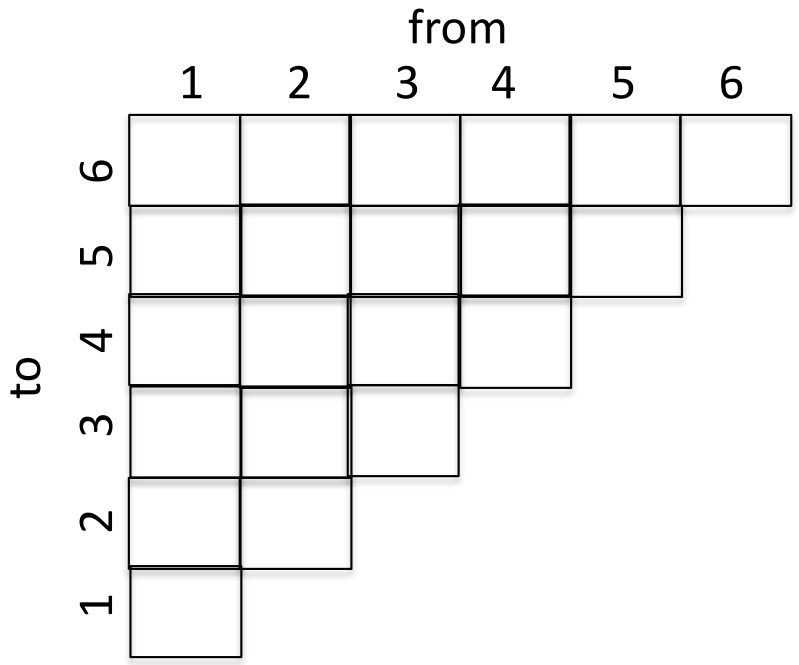

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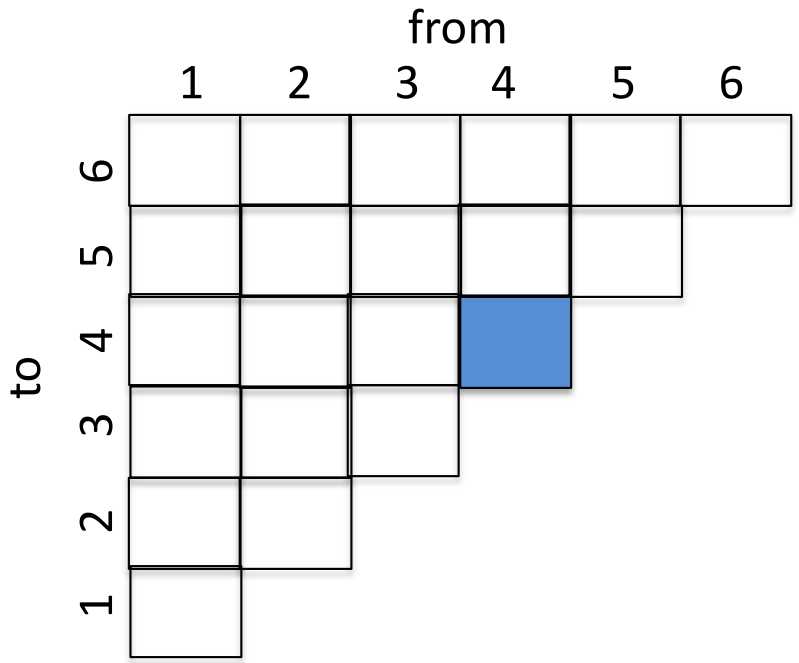

(30, 35, 15, 5, 10, 20, 25)

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What goes in the colored cell?

A. 0
B. 10
C. 200
D. Cannot be determined
E. None of the above

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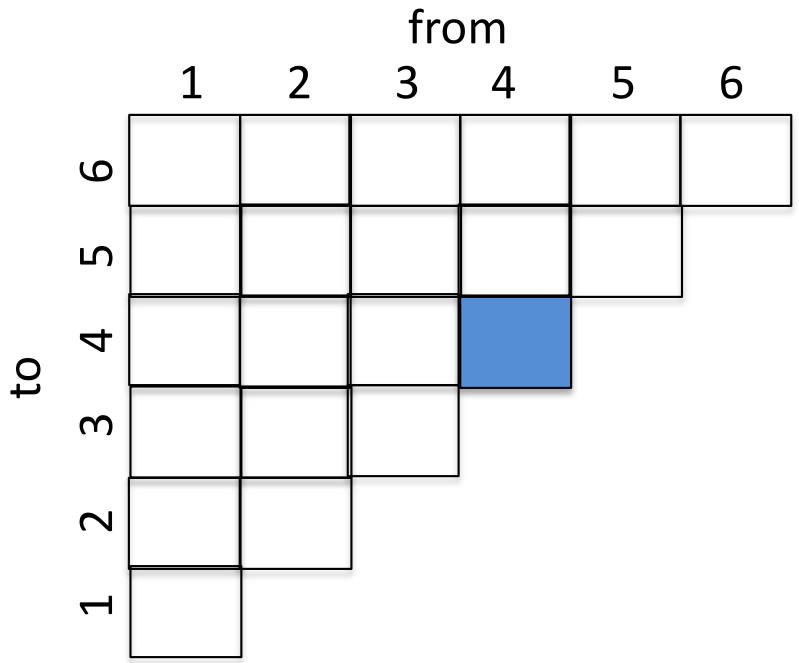

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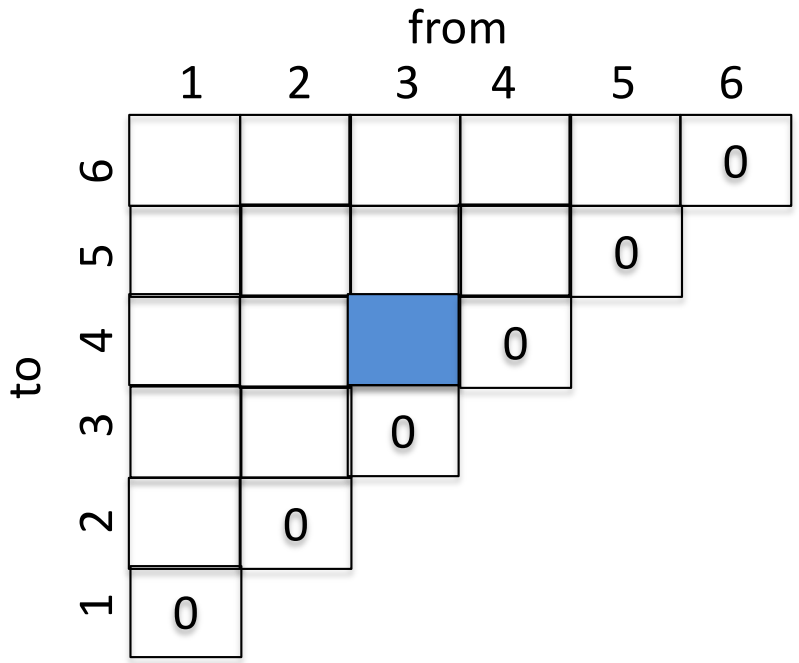

(30, 35, 15, 5, 10, 20, 25)

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What goes in the colored cell?

A. 0
B. 50
C. 750
D. 1000
E. None of the above

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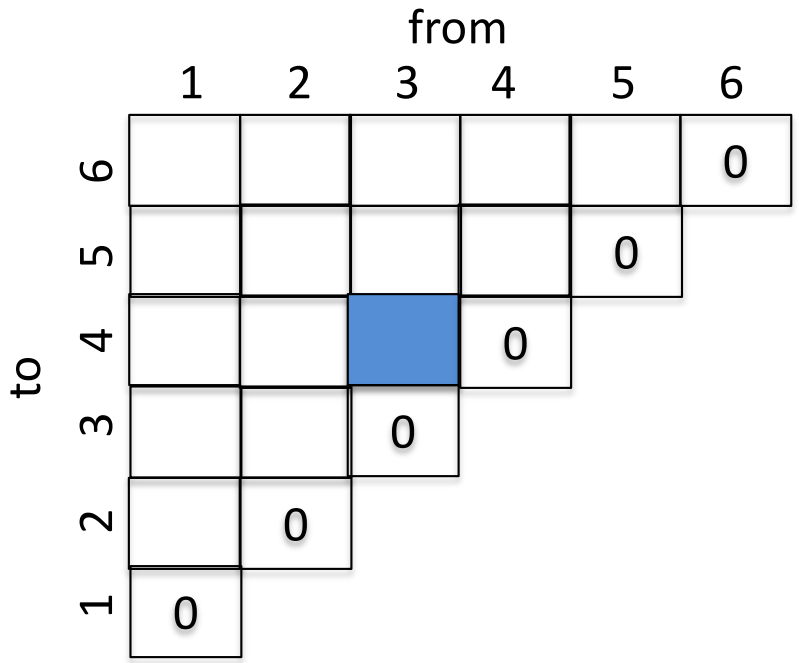

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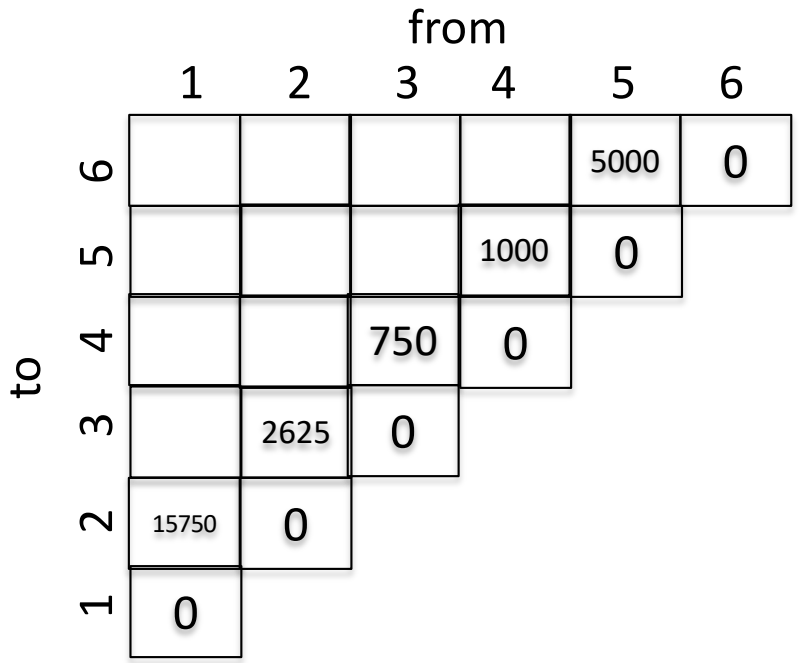

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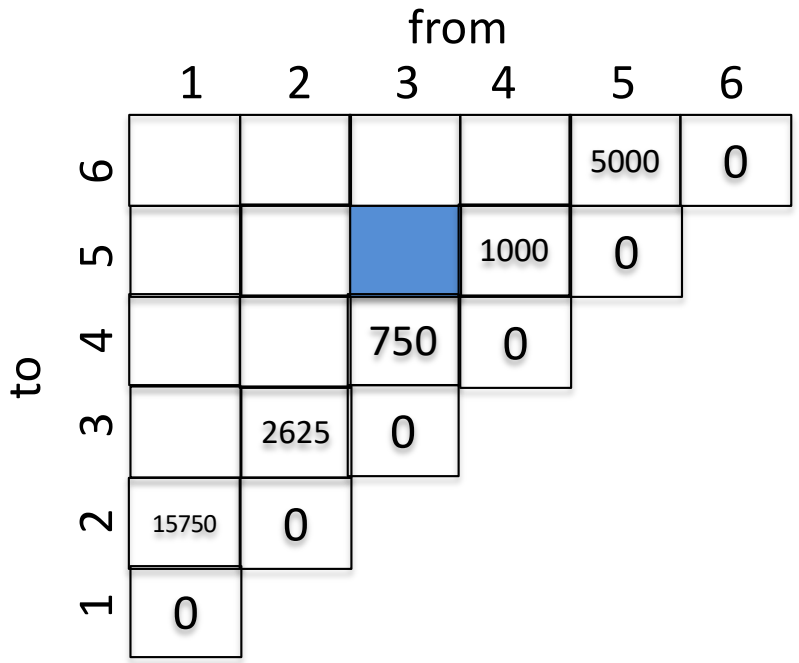

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What goes in the colored cell?

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B. 1000
C. 2500
D. 3750
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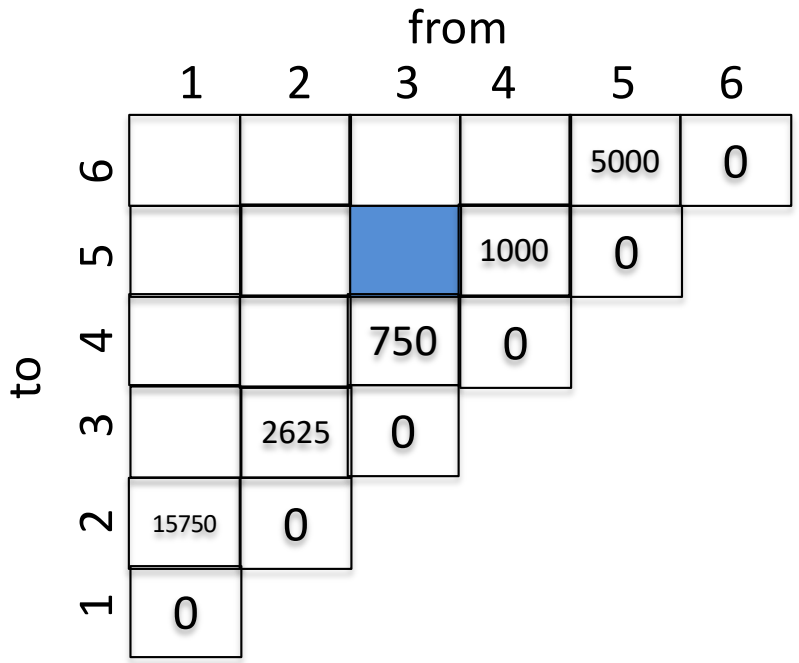

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to \ from	1	2	3	4	5	6
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5	11875	7125	2500	1000	0	
4	9375	4375	750	0		
3	7875	2625	0			
2	15750	0				
1	0					

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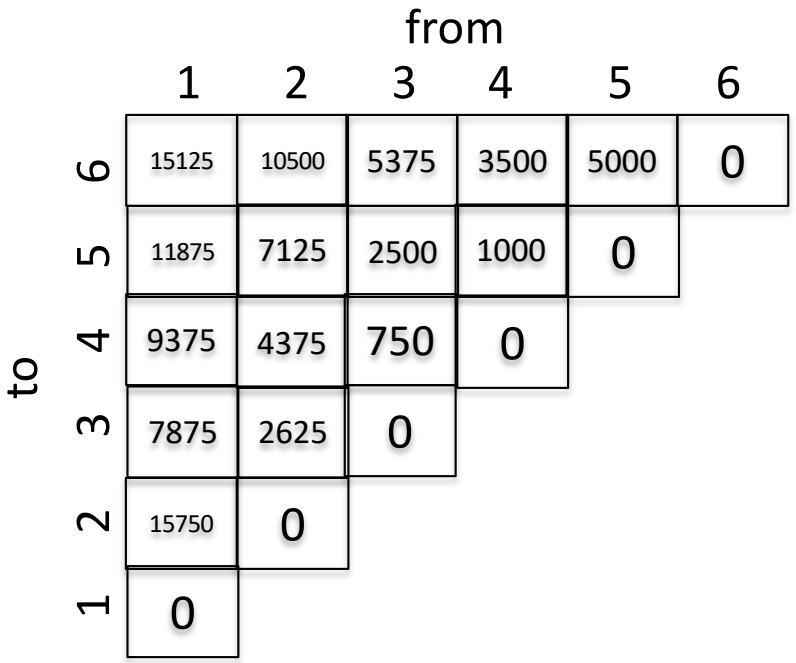

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What is the running time of this algorithm?

A. ~~Θ~~(n)
B. ~~Θ~~(n^2)
C. ~~Θ~~$(n^2 \log n)$
D. ~~Θ~~(n^3)
E. None of the above

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