Graph applications and computing an MST

10/11/24

Administrivia

- HW 4 (Dynamic programming and Graph applications) due Tuesday night

Graph algorithms/problems so far

- BFS: Traverse and find distance in unweighted graph
- DFS: Traverse and handle multiple components
- MST: Find minimum spanning tree (connect at minimum cost)
- Shortest path

Which of the following algorithms is most appropriate for the Skyscraper floors problem?

A. BFS
B. DFS
C. Either BFS or DFS
D. MST
E. Shortest path

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Which of the following algorithms is most appropriate for the Domino game problem?

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C. Either BFS or DFS
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B. DFS
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E. Shortest path

Which of the following algorithms is most appropriate for the Highways problem?

A. BFS
B. DFS
C. Either BFS or DFS
D. MST
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B. DFS
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D. MST
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Which of the following algorithms is most appropriate for the Monitoring the Amazon problem?

A. BFS
B. DFS
C. Either BFS or DFS
D. MST
E. Shortest path

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A. BFS
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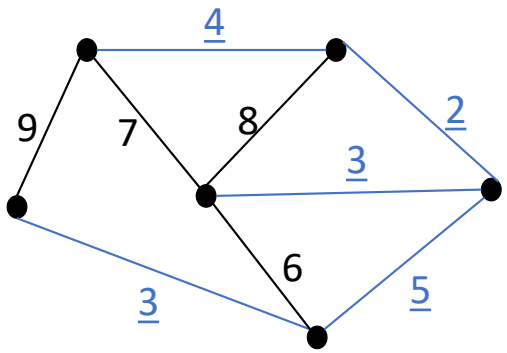
C. Either BFS or DFS
D. MST
E. Shortest path

Recall: Minimum Spanning Tree (MST)

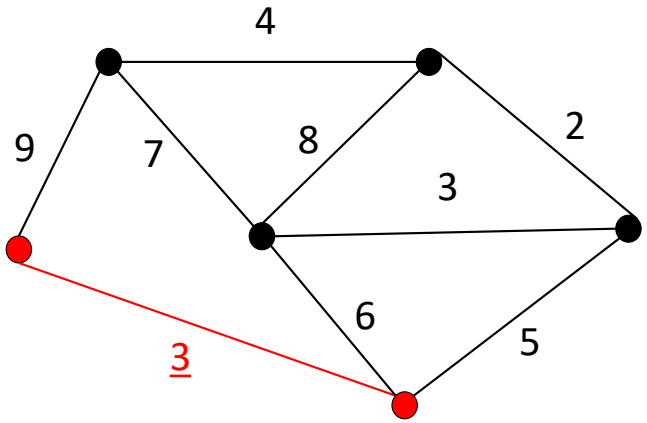
- A subset of edges that connects all vertices and has minimum total weight



Total cost: 17

Prim-Jarnik algorithm

- Repeatedly add the cheapest edge leaving the set of vertices reachable from a root vertex r



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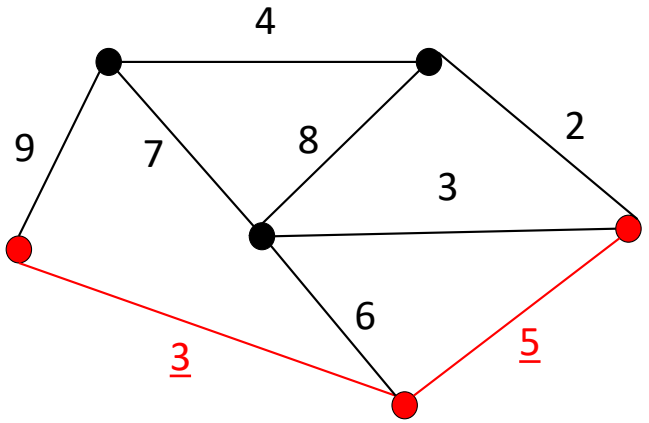

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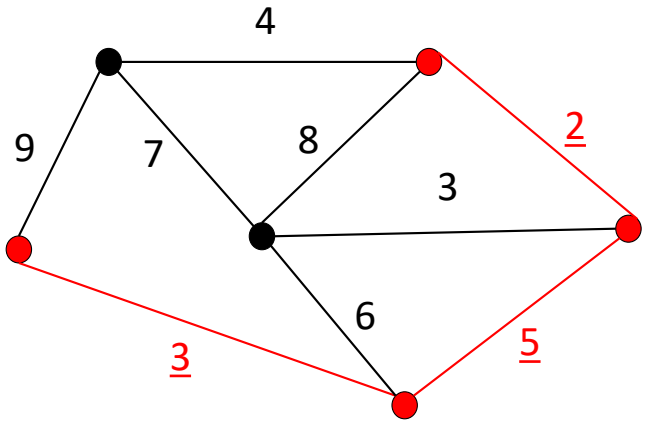


Prim-Jarnik algorithm

- Repeatedly add the cheapest edge leaving the set of vertices reachable from a root vertex r

```
MST-Prim(G,w, r)
        set key of r to 0, set keys for other vertices to ∞
        priority queue Q = G.V
        while Q != ∅
                u = Extract-min(Q)
                for each neighbor v of u
                        if v in Q and w(u,v) < v.key
                                v.key = w(u,v)
```

Cut property

Let G=(V,E) be a connected undirected graph with non-negative edge weights. If A is a subset of E contained in a MST, (S,V-S) is a cut respecting A, and e is one of the cheapest edges from S to V-S, then there is a MST containing A+e.

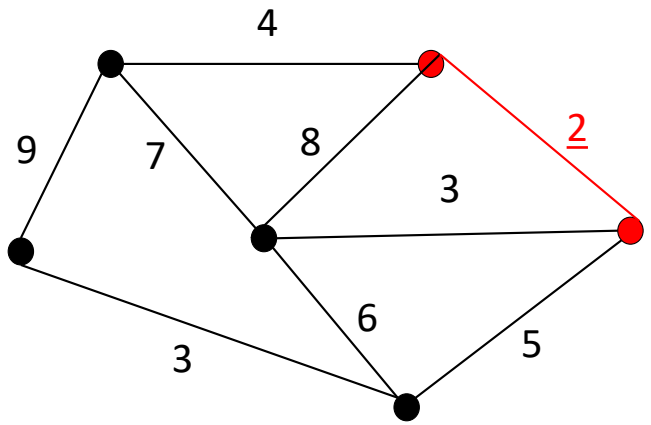
Applying the cut property

Let G=(V,E) be a connected undirected graph with non-negative edge weights. If A is a subset of E contained in a MST, (S,V-S) is a cut respecting A, and e is one of the cheapest edges from S to V-S, then there is a MST containing A+e.

Prim-Jarnik: Repeatedly add the cheapest edge leaving the set of vertices reachable from a root vertex r

Kruskal's algorithm

- Repeatedly add the cheapest edge whose endpoints aren't already connected



Kruskal's algorithm

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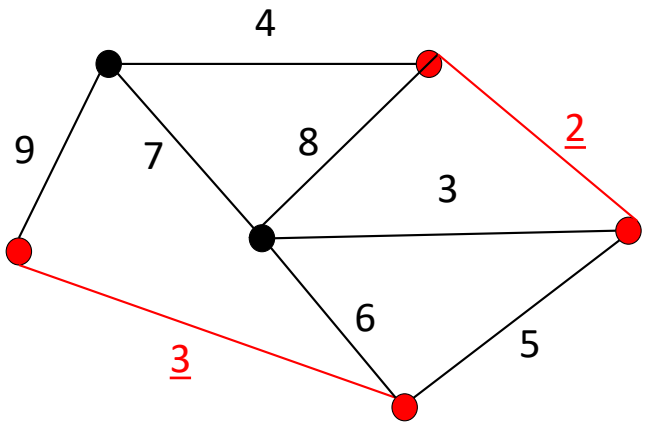

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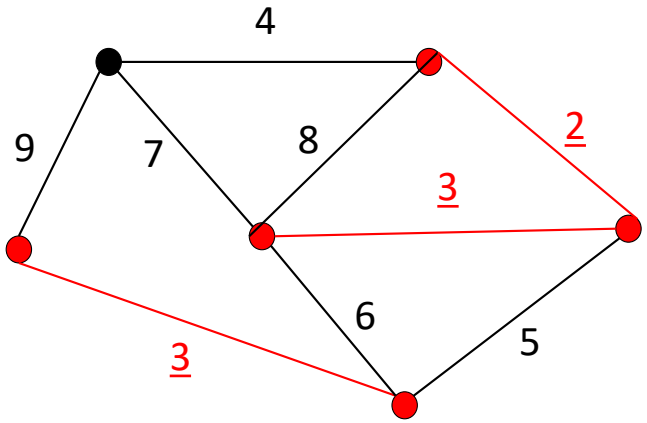

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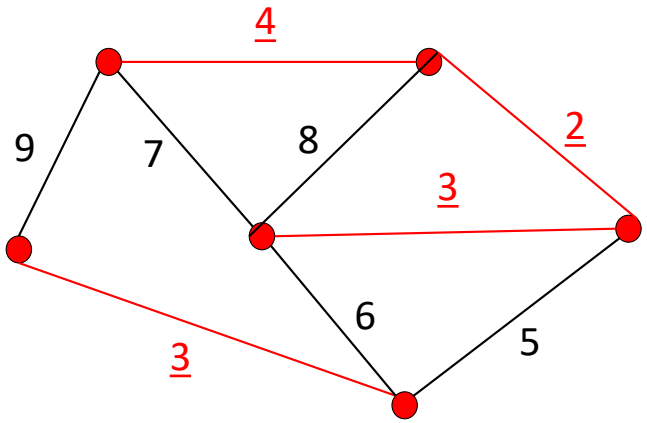

Kruskal's algorithm

- Repeatedly add the cheapest edge whose endpoints aren't already connected

```
MST-Kruskal(G,w)
        A = empty set
        for each vertex v in G.V
                Make-set(v)
        sort edges of G.E into nondecreasing order
        for each edge (u,v) in this order
                if Find-set(u) != Find-set(v)
                        A = A + (u,v)
                        Union(u,v)
        return A
```

Kruskal's algorithm

- Repeatedly add the cheapest edge whose endpoints aren't already connected

Cut property: Let G=(V,E) be a connected undirected graph with non-negative edge weights. If A is a subset of E contained in a MST, (S,V-S) is a cut respecting A, and e is one of the cheapest edges from S to V-S, then there is a MST containing A+e.