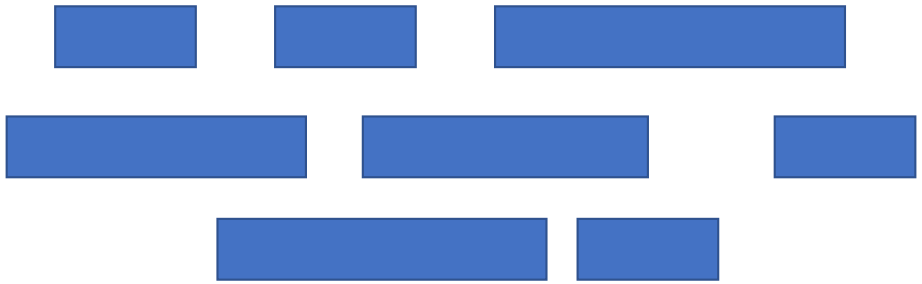
Greedy Algorithms

Greedy algorithms

- Use a simple rule to pick part of the solution, generally in a locally-best way
- Then, prune choices this makes impossible and repeat
- Greedy algorithms don’t always work, but they do for some problems

Application: Activity selection

- Set of n proposed activities $\{a_1, a_2, \ldots, a_n\}$
  - Activity a_i has start time s_i and finish time f_i
- Every activity wants to use a shared resource (e.g. a room) so only one can be scheduled at a time
- Goal: Select a largest possible subset of activities that occur in non-overlapping times



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Does the greedy rule of selecting the activity with the earliest starting time work? (Either sketch a proof or give a counter-example.)

A. Yes

B. No

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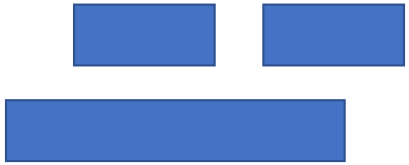

Does the greedy rule of selecting the activity with the shortest interval work? (Either sketch a proof or give a counter-example.)

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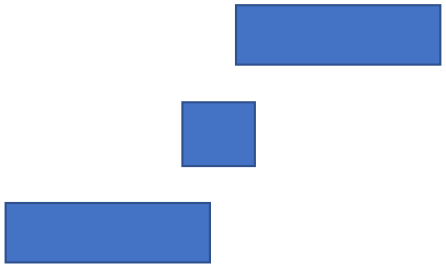

Does the greedy rule of selecting the activity whose interval intersects the fewest other intervals work? (Either sketch a proof or give a counter-example.)

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