Reductions and hardness

How not to handle a problem you can’t solve

“I can’t find an efficient algorithm, I guess I’m just too dumb.”

Image: Garey and Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, 1979

What you'd like to do

"I can't find an efficient algorithm, because no such algorithm is possible!"

Image: Garey and Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, 1979

The next best thing

"I can't find an efficient algorithm, but neither can all these famous people."

Image: Garey and Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, 1979

Decision problems

- Answer is yes or no

- Examples:
  - Given a graph G and an integer k, does G have a spanning tree of cost <= k?
  - Given a set of jobs and integer k, can the jobs be scheduled with maximum lateness <= k?
  - Given a graph G and an integer k, does G have a flow of size >= k?

Reduction

- Showing how to solve an instance of one problem using an algorithm for another problem

- If A is reducible to B ($A \leqslant B$) if an instance x of A can be transformed into an instance f(x) of B such that x in A if and only if f(x) in B

- Examples:

  Median finding $\leqslant$ Sorting

  Bipartite matching $\leqslant$ Network flow

Poly-time reducibility

- We're particularly interested in reductions that take at most polynomial time:

  $A \leqslant_p B$ if x in A if and only if f(x) in B and

  f can be computed in polynomial time

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- If B can be solved in polynomial time, then so can A
- If A cannot be solved in polynomial time, then neither can B

CIRCUIT-SAT

{<C> : C is a satisfiable boolean circuit }

Is the circuit below satisfiable?

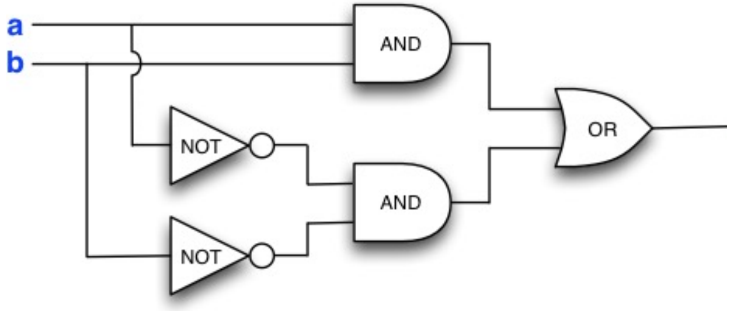
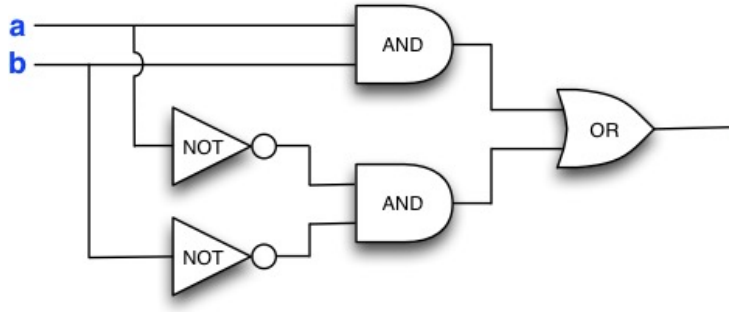


Image: https://algorithms.cs.aalto.fi/Teaching/CS-A1120/2018/notes/round-combinational-logic.html

A. Yes

B. No

Is the circuit below satisfiable?



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A. Yes

B. No