NP-completeness

11/1/24

Administrivia

- HW 6 (flow and greedy algorithms) due Wednesday night
- HW 7 due at end of term (a week from Tuesday)
  - Amortized analysis, NP-completeness
- Final out immediately after that
  - Due at end of finals (10pm, Sunday 11/17)
  - Comprehensive, but weighted toward last part of the course

Hardness so far

- Polynomial-time reductions:

  A $\leqslant_p$ B if x in A if and only if f(x) in B and

  f can be computed in polynomial time

CIRCUIT-SAT $\leqslant_p$ SAT $\leqslant_p$ 3SAT $\leqslant_p$ CLIQUE $\leqslant_p$ INDEPENDENT SET

Complexity classes

- P: Problems that can be decided in polynomial time
- NP: Problems that can be verified in polynomial time

NP-completeness

- A is NP-complete if
  - A in NP
  - L $\leqslant_p$ A for every L in NP

NP-completeness

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- NP-complete problems are the hardest problems in NP
  - If any NP-complete problem is in P, then so is every problem in NP and P = NP
  - If P != NP, then no NP-complete problem can be solved in polynomial time

Cook-Levin Th: CIRCUIT-SAT is NP-complete

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Proof sketch:

- Verification routine can be written as program that runs in poly-time
- Computer updating its state for 1 time step is a circuit
- “Unrolling” this and taking the certificate as inputs gives a circuit that is satisfiable if and only if the original problem instance is a “yes” instance

Showing other problems are NP-complete

- Show they are in NP
- Pick “favorite” NP-complete problem and show how to solve it using the new problem

VERTEX COVER

- A vertex cover is a set of vertices such that every edge has at least one of its vertices in the set

- Given graph G and integer k, does G have a vertex cover of size k?

What is the size of the smallest vertex cover in the graph below?

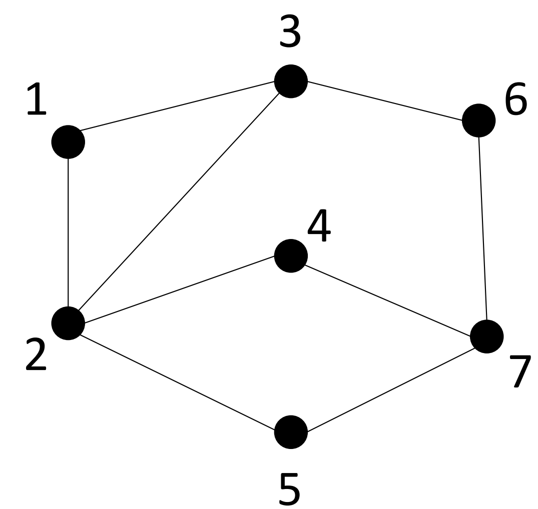
A. 2

B. 3

C. 4

D. 5

E. 6



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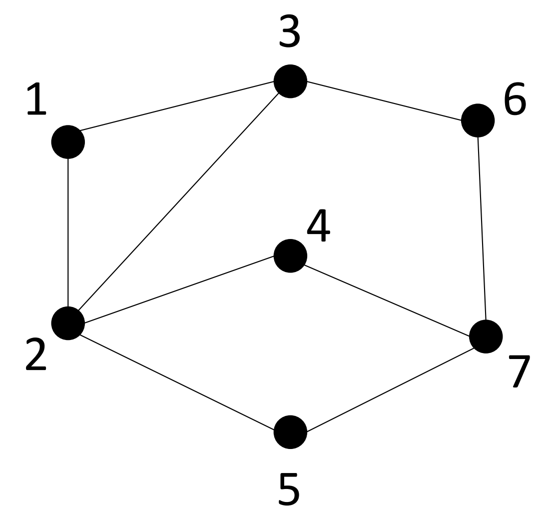


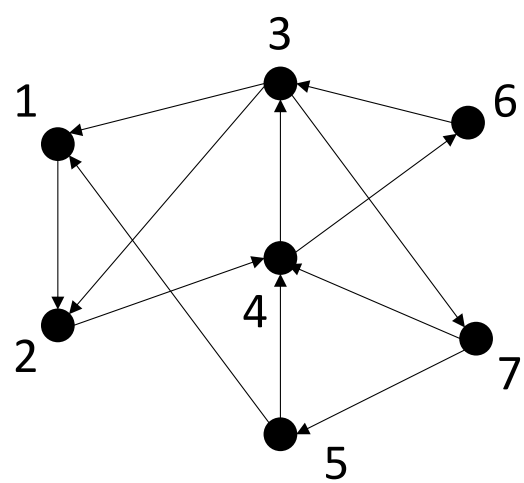
SET COVER

- Given a set U and a collection of other sets S_1, S_2, …, S_n, are there k of the sets whose union is U?

- Example:
  - U = {1, 2, 3, 4, 5, 6}
  - S_1 = {1, 4, 5}, S_2 = {1, 2, 5, 6}, S_3 = {2, 4}, S_4 = {3, 6}
  - "no" for k=2, "yes" for k=3

HAMILTONIAN CYCLE (HAM CYCLE)

- Given a directed graph G, is there is a cycle visiting all the vertices?



HAMILTONIAN PATH (HAM PATH)

- Given a directed graph G, is there is a path visiting all the vertices?