More NP-completeness

11/4/24

Administrivia

- HW 6 (flow and greedy algorithms) due Wednesday night
- HW 7 due at end of term (a week from tomorrow)
 - Amortized analysis, NP-completeness
- Final out immediately after that
 - Due at end of finals (10pm, Sunday 11/17)
 - Comprehensive, but weighted toward last part of the course

Hardness so far

- Poly-time reduction: $A \leq_p B$ if x in A if and only if f(x) in B and can compute f in polynomial time
- Complexity classes
 - P: Problems that can be decided in polynomial time
 - NP: Problems that can be verified in polynomial time
- NP complete: Any problem in NP reduces to them
 - Show they are in NP
 - Pick "favorite" NP-complete problem and show how to solve it using the new problem
- NP-complete problems so far:

CIRCUIT-SAT \leq_p SAT \leq_p 3SAT \leq_p CLIQUE \leq_p INDEPENDENT SET \leq_p VERTEX COVER \leq_p SET COVER $\stackrel{\sim}{\searrow}$ HAMILTONIAN CYCLE

Where we were: HAM PATH

• Given a directed graph G, is there is a path visiting all the vertices?



HAMILTONIAN PATH (HAM PATH)

• Given a directed graph G, is there is a path visiting all the vertices?



• LONGEST PATH: Given G and an integer k, does G have a path of length k?

https://www.youtube.com/watch?v=a3ww0gwEszo

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- B. CLIQUE
- C. INDEPENDENT SET
- D. HAMILTONIAN CYCLE
- E. HAMILTONIAN PATH

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- B. CLIQUE
- C. INDEPENDENT SET
- D. <u>HAMILTONIAN CYCLE</u>
- E. HAMILTONIAN PATH

• Given weighted complete graph and integer k, is there a cycle that visits all the vertices with total cost at most k?

https://xkcd.com/399/

Is there a subset of {6, 9, 12, 13, 14, 18, 21, 22, 34, 40} summing to 100?

- A. Yes
- B. No

Is there a subset of {6, 9, 12, 13, 14, 18, 21, 22, 34, 40} summing to 100?

- A. <u>Yes</u> 12+14+18+22+34=100
- B. No

- Simplifying assumptions:
 - No clause contains both a variable and its negation
 - Every variable appears in at least one clause

 $(x_1 \lor \overline{x_2} \lor \overline{x_3}) \land (x_1 \lor \overline{x_2} \lor x_3)$

| | x_1 | <i>x</i> ₂ | <i>x</i> ₃ | clause 1 | clause 2 |
|---|-------|-----------------------|-----------------------|----------|----------|
| slack for each clause choice for each variable | 1 | 0 | 0 | 1 | 1 |
| | 1 | 0 | 0 | 0 | 0 |
| | 0 | 1 | 0 | 0 | 0 |
| | 0 | 1 | 0 | 1 | 1 |
| | 0 | 0 | 1 | 1 | 0 |
| | 0 | 0 | 1 | 0 | 1 |
| | 0 | 0 | 0 | 1 | 0 |
| | 0 | 0 | 0 | 2 | 0 |
| | 0 | 0 | 0 | 0 | 1 |
| | 0 | 0 | 0 | 0 | 2 |
| Target: | 1 | 1 | 1 | 4 | 4 |