More network flow

10/17/24

Administrivia

- HW 5 (graph algorithms) is due tomorrow
- Exam 2 is out Saturday
  - more multithreaded, dynamic programming, graph algorithms, network flow
  - Same general format (multi-day takehome with open written material and course webpage, closed everything else)
  - No class Monday

Recall: Network flow

- Directed graph with distinguished vertices s (the source) and t (the sink)
- Edge (u,v) has capacity c(u,v)

- Flow f is function E -> numbers obeying:
  - Capacity: 0 <= f(u,v) <= c(u,v) for every edge (u,v)
  - Conservation: sum_u f(u,v) = sum_u f(v,u) for v != s,t

- We've been assuming that flow is integral if all capacities are integers

Exercise 24.1-7

Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is, each vertex v has a limit l(v) on how much flow can pass through v. Show how to transform a flow network G=(V,E) with vertex capacities into an equivalent flow network G'=(V',E') without vertex capacities such that the maximum flow of G' is the same as the maximum flow of G.

K&T Exercise 7.4

Let G be an arbitrary flow network, with a source s, sink t, and an integer capacity for every edge. If flow f is a maximum s-t flow, is it guaranteed to saturate every edge out of s? (i.e. to assign each edge a flow equal to its capacity)

A. Yes

B. No

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A. Yes

B. No

Basic idea for finding max flow

Initialize flow to 0

while there is an augmenting path p from s to t,

    augment flow by minimum capacity along p

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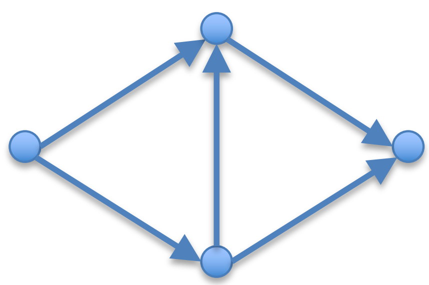

Ford-Fulkerson algorithm

Initialize flow to 0

while there is an augmenting path p from s to t,

    augment flow by minimum capacity along p

    create residual graph

Cuts

- A *cut* is a partition of the vertices (A,B) with $s \in A$ and t $\in B$

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- *Capacity of a cut*:

  c(A,B)=sum of edges crossing it from A to B

Max-flow min-cut Theorem: Capacity of the min cut = value of max flow

Max-flow min-cut Theorem: Capacity of the min cut = value of max flow

Part 1: For any flow f and cut (A,B), v(f) <= c(A,B)

$$
\begin{aligned}
v(f) \quad &= \quad f^{out}(A) - f^{in}(A) \\
&\leq f^{out}(A) \\
&\leq \textstyle\sum_{e \text{ out of } A} c(e) \\
&= \quad c(A,B)
\end{aligned}
$$

Max-flow min-cut Theorem: Capacity of the min cut = value of max flow

Part 2: Ford-Fulkerson finds flow f such that v(f) = c(A,B) for a cut (A,B); this must be the min cut

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Consider last residual graph.

Let A = vertices reachable from s and B = V – A.

$$
\begin{aligned}
v(f) &= f^{out}(A) - f^{in}(A) \\
&= \textstyle\sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e) \\
&= \textstyle\sum_{e \text{ out of } A} c(e) - 0 \\
&= c(A,B)
\end{aligned}
$$

K&T Exercise 7.5

Let G be an arbitrary flow network, with a source s, sink t, and an integer capacity for every edge. Suppose (A,B) is a minimum s-t cut. Now suppose we add 1 to every edge capacity. Is (A,B) guaranteed to be a minimum s-t cut for the new network?

A. Yes

B. No

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A. Yes

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