

$$F(0) = 1$$

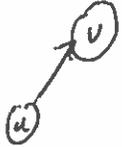
$$F(i+1) = 2^{F(i)} \quad \text{"tower"}$$

$$\log^* N = \text{least } i \text{ st. } F(i) \geq N$$

$$= \text{least } i \text{ st. } \underbrace{\log \log \log \dots \log}_i N \leq 1$$

rank(u) =

Lemma: There are $\leq \frac{N}{2^i}$ nodes of rank i

Lemma:  implies $u.\text{rank} < v.\text{rank}$

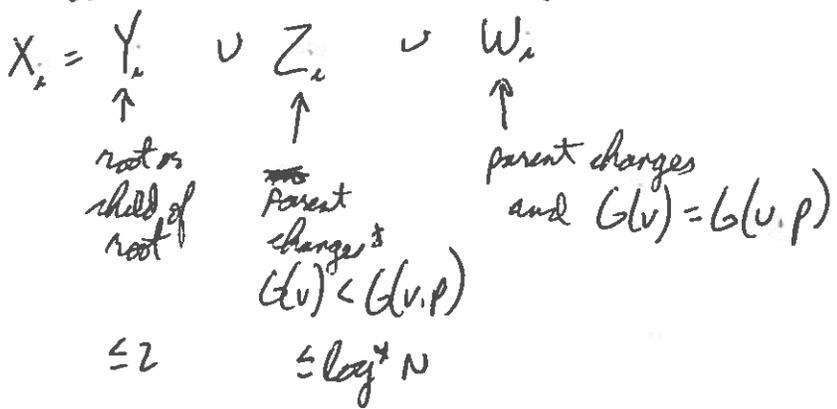
$$G(u) = \text{Group}(u) = \log^*(\text{rank}(u))$$

$$\text{Lemma: } G(u) \leq \log^* \frac{N}{2} \quad \forall u$$

Th: Total time for M operations on N items is $M \log^* N$

Pf: focus on finds

X_i = all nodes on a path during find



$$\sum_{\text{finds}} |W_i|$$

$$= \sum_{\text{nodes } v} \# \text{ finds s.t. } v \in W_i$$

rank(parent) increases each time

$$\leq \sum_{g=0}^{\log^* N} (\# \text{ nodes in group } g) F(g)$$

nodes for a particular group:

$$\sum_{r=0}^{F(g)} \frac{N}{2^r} \leq \frac{N}{2^{F(g)-1}} (1 + \frac{1}{2} + \frac{1}{4} + \dots) = \frac{N}{F(g)}$$

$$\leq \sum_{g=0}^{\log^* N} \frac{N}{F(g)} F(g) = O(N \log^* N) = O(m \log^* N)$$