Physics 167 – Astronomy

Lab Project 1: Measurement of Latitude and Circumference of the Earth

Introduction

You will measure the circumference of the earth using a method first performed by Eratosthenes in 240 B.C. Eratosthenes learned that the sun is directly overhead at noon on the summer solstice in Syene. He measured the length of the shadow cast by a vertical rod in Alexandria at noon on the summer solstice. From his measurement, and assuming the Earth is spherical, Eratosthenes found his latitude. From the known distance to Syene, about 800 km south of Alexandria, Eratosthenes calculated the circumference of the Earth—an amazing accomplishment, especially given the simplicity of his apparatus. In this project, you'll be able to follow in the footsteps of the great Eratosthenes'.

Procedure

Using a borrowed meterstick, measure the length of its shadow on level ground when the sun is highest in the sky (around 1:07)—this will be the time when the shadow has minimum length. Pick the first sunny day you can—the closer you are to the Vernal Equinox (March 20), the more accurate your data will be. This measurement may be most easily done with a partner—one person holds the meterstick and judges that it is vertical while the other marks the shadow position. Record the date, time of day, shadow length, and where you made the measurement. Also estimate and record the uncertainty in your measurement of the shadow's length (use your best judgment on this—it is an estimate, not an exact value).

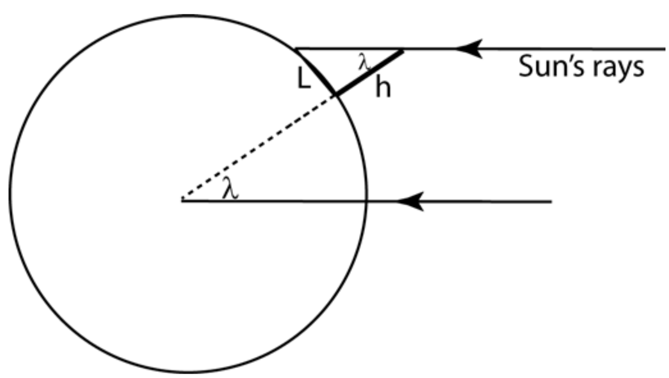


Figure 1. h = height of vertical rod, L = length of shadow, λ = latitude angle.
Note that $\tan \lambda = h/L$.

Questions

1. a. Present your data in a neat table, including the date, time of day, shadow length and uncertainty, and where you made the measurement.
b. Calculate the latitude λ of Galesburg from your measurement. See Figure 1 for a hint.

c. Compare your answer with the accepted value (google it!) What is the percent error in your measurement?

d. Figure out the uncertainty in your latitude measurement. This is the difference between (1) latitude you got using your measured shadow length, and (2) the latitude you find using the worst case shadow length (measured length + uncertainty in length). What is the percent uncertainty in your latitude measurement? Is your result for Galesburg's latitude reasonable, given the uncertainty in your measurement?

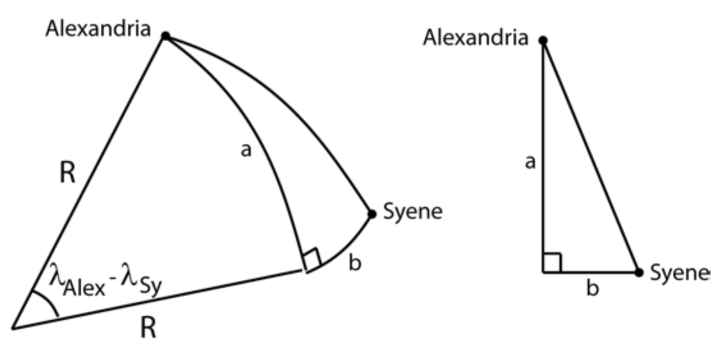
2. a. Given that the distance from Galesburg to the equator is 4558 km, calculate the circumference of the Earth in km.

Hint: Recall that arc length s on a circle is given by $s = R\,\lambda$. Here s = 4558 km and λ is the latitude angle in radians ($360° = 2\pi$ radians). So find the radius R of the Earth, and then the circumference is $2\pi R$.

b. Compare your answer with the accepted value, and report the percent error.

3. Redraw Figure 1, and also show (1) the equator, (2) Syene, and (3) Alexandria, if the Figure illustrates Erastosthenes' experiment, performed on the summer solstice.

4. Look up the latitude and longitude of Alexandria and Aswan (the modern name of Syene) in Egypt. Given that the radius of the Earth is 6378 km, estimate the north-south distance and the east-west distance between the two cities. If Eratosthenes used the direct distance between cities (rather than the north-south distance), approximately what percent error would this cause in his calculation of the Earth's circumference?



Hint: Since the triangle shown in Figure 2 is small, you can assume it is flat and use regular plane geometry (instead of spherical trigonometry!) In that case, the Pythagorean theorem gives the distance between Alexandria and Syene if you know the two legs of the right triangle. The north-south leg a is given by $a = R(\lambda_{Alexandria} - \lambda_{Syene})$, where R is the Earth's radius and the λ's are the latitudes of the two cities in radians. Remember that $360° = 2\pi$ radians.